



MASENO SCHOOL

Topic 11: Sequence & Series

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Important Formulas

- ✓ The n^{th} term in an arithmetic progression (A.P): $T_n = a + (n-1)d$ where a is the first term and d is the common difference.
- ✓ The sum of the first n terms in an arithmetic series: $S_n = \frac{n}{2}(a+l)$ where $l = T_n$, hence $S_n = \frac{n}{2}\{2a + (n-1)d\}$
- ✓ The n^{th} term geometric progression (G.P): $T_n = ar^{n-1}$ where a is the first term and r is the common ratio.
- ✓ The sum of the first n terms in geometric series: $S_n = \frac{a(r^n - 1)}{r - 1}$; $r > 1$ or use $S_n = \frac{a(1 - r^n)}{1 - r}$; $r < 1$

Section I Questions

1. Find the sum of the series $-22 + -19 + -16 + \dots + 239$
2. Find the sum of the series $8 + 4 + 2 + \dots + \frac{1}{512}$
3. The sum of n terms of the sequence: 3, 9, 15, 21 ... is 7500. Determine the value of n .
4. The exterior angles of a hexagon form an arithmetic progression. If the smallest angle is 15° , find the size of the largest angle of the hexagon.
5. An arithmetic progression whose first term is 2 and whose n^{th} term is 97, has the sum of its first n terms equal to 990. Find n and the common difference.
6. Every time an insect jumps forward, the distance covered is half of the previous jump. If the insect initially jumped 8.4 cm, calculate:
 - (a) The distance of the 8th jump.
 - (b) The total distance covered after the 8th jump.
7. The 11th term of an arithmetic progression is four times the 2nd term. If the sum of the first seven terms of the arithmetic progression is 175, find the first term and the common difference.
8. The 2nd, 4th and 7th terms of an A.P are the first three consecutive terms of a G.P. If the common difference of the A.P is 2, find:
 - (a) The common ratio of the G.P
 - (b) The sum of the first 8 terms of the G.P.
9. In a G.P, the sum of the second and the third terms is 12, and the sum of the third and the fourth terms is -24 . Find the first term and the common ratio.
10. The 1st, 2nd and the 3rd terms of a sequence are given as $\{4+5+6\}$, $\{7+8+9\}$, and $\{10+11+12\}$ respectively. Find the 20th term.
11. The first, the third and the seventh terms of an increasing arithmetic progression (A.P) are three consecutive terms of a geometric progression (G.P). If the first term of the A.P is 10. Find the common difference of the A.P and the common ratio of the G.P.
12. The sum of the first four terms of an arithmetic progression (A.P) is 14. If the sum of the first eight terms is 108, find the sixth term of the progression.
13. The difference between the 8th term and the 4th term of an arithmetic progression is 24. The first term exceeds the common difference by 2. Find the sum of the first 10 terms of the progression.

14. The seventh term of an arithmetic sequence is 17. Three times the third term is 3. Calculate the first term and the common difference of the sequence.
15. The third and the tenth terms of an arithmetic progression are 11 and 39 respectively. Find:
 - (a) The first term and the common difference of the progression.
 - (b) The sum of the first 50 terms of the progression.
16. The third and the sixth terms of a geometric progression are -64 and 8 respectively. Find the common ratio and the first term of the geometric progression
17. The 10th, 25th and the last term of an arithmetic progression are 313, 193 and -7 . Find the number of terms in the progression.
18. A geometric progression is such that the sum of its first three terms is 14. If the first term 2, find the possible values of its common ratio.
19. A ball is dropped from a height of 30 metres. Each time it strikes the ground, it bounces up to 0.8 of the previous height. How many times does the ball need to strike the ground before its bounce is less than 3 metres?
20. Initially, a pendulum swings through an arc of 50 cm. On each successive swing, the length of the arc is decreased by 10%. After 10 swings, what total length will the pendulum have swung?

Section II Questions

1. Alice and Brian were employed at the beginning of the same year in a five year contract. Their annual salaries in Shillings progressed as follows:
Alice: 330000, 356400, 382800, ...
Brian: 330000, 356400, 384912, ...
 - (a) Calculate:
 - (i) Alice's monthly salary in her fifth year of employment. (3 marks)
 - (ii) Brian's annual percentage rate of salary increment and hence write down an expression for his monthly salary in his n^{th} year of employment. (3 marks)
 - (b) Calculate the difference in the total income of Alice and Brian by the end of their contract. (4 marks)
2. The first, fifth and seventh terms of an Arithmetic Progression (A.P) correspond to the first three consecutive terms of a decreasing Geometric Progression (G.P). The first term of each progression is 64, the common difference of the A.P is d and the common ratio of the G.P is r .
 - (a) Write two equations involving d and r hence calculate the values of d and r . (4 marks)
 - (b) Find the sum of the first 24 terms of the Arithmetic Progression (A.P) (3 marks)
 - (c) Find the number of terms, n , for which the sum the Geometric Progression (G.P) is $127\frac{7}{8}$ (3 marks)
3. An arithmetic progression (A.P) has the first term as a and the common difference as d .
 - (a) Write in terms of a and d , the 3rd, 9th and 25th terms of the progression. (1 mark)
 - (b) The progression is increasing and the 3rd, 9th and 25th terms form the first three consecutive terms of a geometric progression (G.P). If the sum of the first 8 terms of the A.P is 153, calculate:
 - (i) The first term and the common difference of the A.P. (4 marks)
 - (ii) The sum of the first 20 terms of the A.P (2 marks)
 - (iii) Find the sum of the first 5 terms of the G.P. (3 marks)
4. The 1st, the 7th and 25th terms of an arithmetic progression are the first three consecutive terms of a geometric progression. The 20th term of the arithmetic progression is 44. Find:
 - (a) The first term and common difference of the arithmetic progression. (3 marks)
 - (b) The number of terms of the arithmetic progression for which the sum of the terms is 1800. (3 marks)
 - (c) The 10th term of geometric progression. (2 marks)
 - (d) The sum of the first n terms of the geometric progression. (2 marks)

5. A geometric progression (G.P.) is such that the product of its first three terms is 8,000.
- (a) Taking the first term as a and the common ratio as r , express r in terms of a . (2 marks)
 - (b) The sum of the first three terms in (a) above is 78.
 - (i) Determine the first term and the common ratio of two possible sequences. (4 marks)
 - (ii) Hence write the first 4 terms of the two sequences. (2 marks)
 - (c) Find the product of the 8th term of the two sequences. (2 marks)
6. The first three terms of a geometric series are: $2x$, $x-8$ and $2x+5$ respectively for which x is a integer.
- (a) Find:
 - (i) The value of x . (3 marks)
 - (ii) The first term and the common ratio in the series. (2 marks)
 - (b) Calculate:
 - (i) The value of the seventh term. (2 marks)
 - (ii) The number of terms for which the sum is -16.625 . (3 marks)
7. The first term of an increasing arithmetic sequence is $(2x-1)$ and the common difference is x . The product of the first and third term is 21.
- (a) Find the value of x . (3 marks)
 - (b) If the last term of the arithmetic sequence is 105, calculate the sum of the terms. (3 marks)
 - (c) Given that the 1st, the 4th and 13th terms of the arithmetic sequence form the first three consecutive terms of a geometric progression, find:
 - (i) The 12th term of the geometric progression. (2 marks)
 - (ii) The sum of the first 12 terms of the geometric progression. (2 marks)
8. The product of the first three consecutive terms at a geometric progression is 729. If the first term is a and the common ratio is r
- (a) Express r in terms of a . (2 marks)
 - (b) Given that the sum of the three terms above is 39, find the values of a and r hence write down two possible sequences each up to the 4th term. (5 marks)
 - (c) The sum of the first n terms of the sequence is 3279. Find n . (3 marks)
9. The second, sixth and eighth terms of an Arithmetic Progression (A.P) correspond to the first three consecutive terms of an increasing Geometric Progression (G.P). If the first term of the A.P is -36 , the common difference of the A.P is d and the common ratio of the G.P is r :
- (a) (i) Write two expressions of r in terms of d . (2 marks)
 - (ii) Find the values of d and r . (4 marks)
 - (b) Find the sum of the first 10 terms of the G.P. (2 marks)
 - (c) Find the least number of terms of the A.P for which the sum is a positive integer. (2 marks)
10. The sum of the first five terms of an increasing arithmetic progression (A.P) is 50 while the sum of the first six terms of the same progression is 66.
- (a) Find the 1st term and the common difference of the A.P. (3 marks)
 - (b) Calculate the sum of the first 20 terms of the A.P. (2 marks)
 - (c) The first term and two other terms of the A.P form a geometric progression (G.P). If the sum of the three terms in the G.P is 78, calculate:
 - (i) The common ratio of the G.P. (3 marks)
 - (ii) The sum of the first 6 terms of the G.P. (2 marks)

Answers (Section I)

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| 1. 9548 | 8. (a) $r=1.5$ (b) $394\frac{1}{16}$ | 15. (a) $a=3, d=4$ (b) 5050 |
| 2. $15\frac{511}{512}$ | 9. $a=6, d=-2$ | 16. $r=-\frac{1}{2}, a=-256$ |
| 3. 50 | 10. $\{61+62+63\}$ | 17. $a=385, d=-8, n=50$ |
| 4. 225° | 11. $d=5, r=2$ | 18. $r=-3, 2$ |
| 5. $n=20, d=5$ | 12. $a=-4, d=5, T_6=21$ | 19. $n=12$ |
| 6. (a) 0.065625 (b) 16.734375 | 13. 350 | 20. 325.66078 |
| 7. $a=10, d=5$ | 14. $a=-7, d=4$ | |

Answers (Section II)

1. (a) (i) Ksh. 36300 (ii) $\frac{330000 \times 1.08^4}{12}$ (b) Ksh. 21 978.3168
2. (a) $r = \frac{64+4d}{64}, r = \frac{64+6d}{64+4d}, d = -8, r = \frac{1}{2}$ (b) 480 (c) 10
3. (a) $T_3 = a + 2d, T_9 = a + 8d, T_{25} = a + 24d$ (b) (i) $a = 6, d = 3.75$ (ii) 832.5 (iii) $1084\frac{1}{6}$
4. (a) $a = 6, d = 2$ (b) 40 (c) 118098 (d) $S_n = 3^{n+1} - 3$
5. (a) $r = \frac{20}{a}$ (b) (i) $a = 8, r = 2.5$ and $a = 50, r = 0.4$ (ii) 8, 20, 50, 125, ... and 50, 20, 8, 3.2, ... (c) 400
6. (a) (i) $x = 2$ (ii) $a = 4, r = -1.5$ (b) (i) $45\frac{9}{16}$ (ii) -16.625
7. (a) $x = 2$, Sequence: 3, 5, 7, 9, ... (b) $n = 52, S_{52} = 2808$ (c) (i) 531 441 (ii) 797 160
8. (a) $r = \frac{9}{a}$ (b) $a = 3, r = 3$ and $a = 27, r = \frac{1}{3}, 3, 9, 27, 81, \dots$ and $27, 9, 3, 1, \dots$ (c) $n = 7$
9. (a) (i) $r = \frac{5d-36}{d-36}, r = \frac{7d-36}{5d-36}$ (ii) $d = 4, r = 0.5$ (b) $-63\frac{15}{16}$ (c) $n = 19$
10. (a) $a = 6, d = 2$ (b) 500 (c) (i) $r = 3$ (ii) 2184