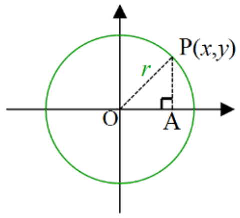
MASENO SCHOOL

Topic 16: EQUATION OF A CIRCLE

By Mr. Patrick Mboya

Introduction



Consider a circle, centre $O(0,0)$ and radius, r units, as shown in the figure alongside. If point $P(x, y)$ is on the circle, then the triangle OAP has $OA = x$ and $AP = y$. By Pythagoras' theorem, we have $x^2 + y^2 = r^2$ which is the equation of the circle centre $O(0,0)$ and radius, r units.

Example 1

Find the equation of a circle whose centre is at the origin and radius = 3.5 units.

Solution

From the equation in the form $x^2 + y^2 = r^2$ where $r = 3.5$, we have $x^2 + y^2 = 12.25$ as the equation of the circle which simplifies to $4x^2 + 4y^2 = 49$.

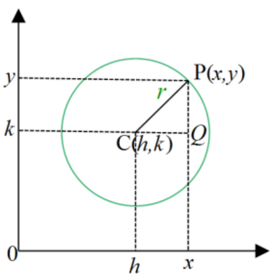
Example 2

Calculate the area of a circle whose equation is $y = \sqrt{(7+x)(7-x)}$. Take $\pi = \frac{22}{7}$.

Solution

$y = \sqrt{(7+x)(7-x)} = \sqrt{49 - x^2}$ hence $y^2 = 49 - x^2$ and thus $x^2 + y^2 = 49$ is the equation of the circle. Its centre is at $O(0,0)$ and the radius is 7 units.

$\therefore$ Area $= \frac{22}{7} \times 7^2 = 154$ sq. units



If the centre of the circle lies at a point $C(h,k)$ as shown in the figure alongside, we have the length $CQ = (x-h)$ and $QP = (y-k)$. By Pythagoras' theorem, we have $(x-h)^2 + (y-k)^2 = r^2$ which is the equation of the circle with centre $C(h,k)$ and radius r units. When we expand and simplify, we obtain $x^2 - 2hx + h^2 + y^2 - 2ky + k^2 = r^2$. We then rearrange the equation to obtain $x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 = 0$. And since h, k and r are constants, we can substitute $-2h = M$, $-2k = N$ and $h^2 + k^2 - r^2 = P$ so that we get $x^2 + y^2 + Mx + Ny + P = 0$ as the standard form of the equation of a circle with centre at $C(h,k)$ and radius r units

Example 3

Find the equation of a circle centre $(2,-3)$ and radius 5 units.

Solution

Since $h = 2$, $k = -3$ and $r = 5$, we substitute these in the equation $(x-h)^2 + (y-k)^2 = r^2$ and obtain $(x-2)^2 + (y+3)^2 = 25$. This simplifies to:

$$x^2 - 4x + 4 + y^2 + 6y + 9 = 25$$

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

#STAYSAFE
STAY AT HOME, SAVE YOUR LIFE:)

Example 4

Find the centre and radius of a circle whose equation is $x^2+y^2+6x-10y-2=0$.

Solution

Collecting like terms, we have $x^2+6x+y^2-10y=2$.

We then complete the squares in x and in y and obtain $x^2+6x+9+y^2-10y+25=2+9+25$ which factorizes to $(x+3)^2+(y-5)^2=36$.

Compare this to $(x-h)^2+(y-k)^2=r^2$ and see that $h=-3,\ k=5$ and $r=6$ units. $\therefore$ Centre $(-3,5),\ r=6$.

Example 5

The equation of a circle is given as $2x^2+2y^2-8x+5y+10=0$. Find the radius of the circle and the coordinates of its centre.

Solution

Expressing this equation in the form $x^2+y^2+Mx+Ny+P=0$, we have $x^2+y^2-4x+\frac{5}{2}y+5=0$.

Collecting like terms, we have $x^2-4x+y^2+\frac{5}{2}y=-5$

We then complete the squares in x and in y and obtain $x^2-4x+4+y^2+\frac{5}{2}y+\frac{25}{16}=-5+4+\frac{25}{16}$ which factorizes to $(x-2)^2+\left(y+\frac{5}{4}\right)^2=\frac{9}{16}$.

Compare this to $(x-h)^2+(y-k)^2=r^2$ and see that $h=2,\ k=-1\frac{1}{4}$ and $r=\frac{3}{4}$ units. $\therefore$ Centre $\left(2,-1\frac{1}{4}\right),\ r=\frac{3}{4}$.

Exercise

1. Determine the equation of a circle centre $\mathrm{M}\left(\frac{1}{2},-\frac{1}{2}\right)$ and radius $r=3$ units. Give your answer in the form $ax^2+by^2+cx+dy+e=0$ where $a,\ b,\ c,\ d$ and e are integers.

2. The points with coordinates $(7,-10)$ and $(-3,8)$ are the end points of a diameter of a circle centre A. determine the equation of the circle in the form $x^2+y^2+ax+by+c=0$ where $a,\ b$ and c are constants.

3. The equation of a circle is given as $4x^2+4y^2-16x+24y+3=0$. Find the centre of the circle and its radius.

4. The equation of a circle is given as $x^2+y^2+4x-5=0$. Find the centre of the circle and its radius.

5. Obtain the centre and the radius of the circle represented by the equation $x^2+y^2-10y+16=0$.

6. Show that $3x^2+3y^2+6x-12y-12=0$ is an equation of a circle hence state the radius and the centre of the circle.

7. Find the centre and the radius of a circle whose equation is given by $4(x^2+y^2)=12(x-y)+7$.

8. Find the centre and the radius of a circle whose equation is given by $2x(x-3)+2y(y+5)+9=0$

9. AB is the diameter of a circle. Given the coordinates of A and B as $(2,-3)$ and $(4,-7)$ respectively, find the equation of the circle in the form $x^2+y^2+ax+by+c=0$ where $a,\ b$ and c are integers.

10. Find the equation of a circle with the end points of a diameter at $(4,3)$ and $(0,1)$. Give your answer in the form in the form $x^2+y^2+ax+by+c=0$ where $a,\ b$ and c are integers.

#STAYSAFE
STAY AT HOME, SAVE YOUR LIFE:)

11. Find the circles that satisfy the conditions: Radius = $\sqrt{17}$, centre on the x – axis, and passes through the point $(0,1)$. Give your answers in the form in the form $x^2+y^2+ax+by+c=0$ where a, b and c are integers.

12. A circle whose centre is at $(1,3)$ has the x – axis as its tangent. Determine the equation of the circle in the form in the form $x^2+y^2+ax+by+c=0$ where a, b and c are integers.

13. Find the equation of the line containing the centres of the two circles $x^2+y^2-4x+6y+4=0$ and $x^2+y^2+6x+4y+9=0$

14. The line $x-2y+4=0$ is a tangent to a circle at $(0,2)$. The line $y=2x-7$ is a tangent to the same circle at $(3,-1)$. Find:

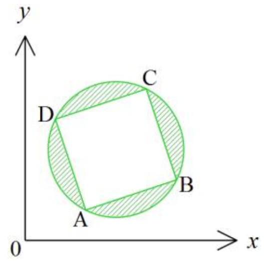
(a) The centre and radius of the circle.

(b) The equation of the circle in the form $x^2+y^2+ax+by+c=0$ where a, b and c are integers.

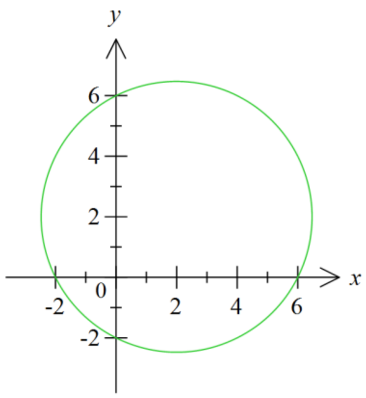
15. The area of an annulus between two concentric circles is 8π square units. If the equation of the larger circle is given as $4x^2+4y^2-8x+8y-73=0$, find the equation of the smaller circle.

16. Calculate the length of the tangent from a point $(-9,9)$ to the circle whose equation is $x^2+y^2+6x-10y-2=0$.

17. In the figure below, ABCD is a square inscribed in a circle whose equation is $x^2+y^2-6x-6y+10=0$. Calculate, in terms of π, the area of the shaded segments.



18. In the figure below, the circle passes through the points $(-2,0)$, $(6,0)$, $(0,-2)$ and $(0,6)$.



Find:

(a) The centre and the radius of the circle.

(b) The equation of the circle in the form $x^2+y^2+ax+by+c=0$ where a, b and c are integers.

19. Given that a circle whose equation is $0.25x^2+0.25y^2+x+ky-1=0$ passes through a point $(-2,4)$, find:

(a) The value of k

(b) The centre and the radius of the circle.

20. Three points $A(3,-1)$, $B(1,4)$ and $C(-4,2)$ lie on a circle. Calculate:

(a) The centre and the radius of the circle.

(b) The equation of the circle in the form $x^2+y^2+ax+by+c=0$ where a, b and c are integers.

#STAYSAFE
STAY AT HOME, SAVE YOUR LIFE:)

Answers

1. $2x^2+2y^2-2x+2y-17=0$

2. $Centre(2,-1),\ r=\sqrt{106},\ x^2+y^2-4x+2y-101=0$

3. $Centre(2,-3),\ r=3.5$

4. $Centre(-2,0),\ r=3$

5. $Centre(0,5),\ r=3$

6. $Centre(-1,2),\ r=3$

7. $Centre\left(1\frac{1}{2},-1\frac{1}{2}\right),\ r=2.5$

8. $Centre\left(1\frac{1}{2},-2\frac{1}{2}\right),\ r=2$

9. $Centre(3,-5),\ r=\sqrt{5},\ x^2+y^2-6x+10y+29=0$

10. $Centre(2,2),\ r=\sqrt{5},\ x^2+y^2-4x-4y+3=0$

11. $Centre(4,0)$ Eqn: $x^2+y^2-8x-1=0$ and $Centre(-4,0)$ Eqn: $x^2+y^2+8x-1=0$

12. $x^2+y^2-2x-6y+1=0$

13. $Centres:(2,-3)$ and $(-3,-2): Eqn: x+5y+13=0$

14. (a) $Centre(1,0),\ r=\sqrt{5}$ (b) $x^2+y^2-2x-4=0$

15. $Centre(1,-1), R=4.5,\ r=3.5,\ Eqn: 4x^2+4y^2-8x+8y-41=0$

16. $Centre(-3,5),\ r=6,$ length of the tangent $=4$

17. $8\pi-16$

18. (a) $Centre(2,2),\ r=2\sqrt{5}$ (b) $x^2+y^2-4x-4y-12=0$

19. (a) $k=-\frac{1}{2}$ (b) $Centre(-2,1),\ r=3$

20. (a) $Centre\left(-\frac{1}{2},\frac{1}{2}\right),\ r=\frac{\sqrt{58}}{2}$ (b) $x^2+y^2+x-y-14=0$