MASENO SCHOOL

Topic 12: Vectors II

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Section I Questions

1. Given that $a\underset{\sim}{i}+\frac{1}{3}\underset{\sim}{j}+\frac{2}{3}\underset{\sim}{k}$ is a unit vector, find a.

2. Vector $\underset{\sim}{q}$ has a magnitude of 7 and is parallel to vector $\underset{\sim}{p}$. Given that $\underset{\sim}{p}=3\underset{\sim}{i}-\underset{\sim}{j}+\frac{3}{2}\underset{\sim}{k}$, express $\underset{\sim}{q}$ in terms of $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$.

3. The position vectors of **X** and **Y** are $\underset{\sim}{x}=2\underset{\sim}{i}+\underset{\sim}{j}-3\underset{\sim}{k}$ and $\underset{\sim}{y}=3\underset{\sim}{i}+2\underset{\sim}{j}-2\underset{\sim}{k}$ respectively. Find $|\mathbf{XY}|$.

4. Given that $\underset{\sim}{x}=2\underset{\sim}{i}+\underset{\sim}{j}-2\underset{\sim}{k}$, $\underset{\sim}{y}=-3\underset{\sim}{i}+4\underset{\sim}{j}-\underset{\sim}{k}$ and $\underset{\sim}{z}=5\underset{\sim}{i}+3\underset{\sim}{j}+2\underset{\sim}{k}$ and that $\underset{\sim}{p}=3\underset{\sim}{x}-\underset{\sim}{y}+2\underset{\sim}{z}$, find the magnitude of $\underset{\sim}{p}$ to 3 significant figures.

5. Given that $\underset{\sim}{a}=\underset{\sim}{i}-2\underset{\sim}{j}+3\underset{\sim}{k}$, $\underset{\sim}{b}=-2\underset{\sim}{i}+3\underset{\sim}{j}-\underset{\sim}{k}$ and $\underset{\sim}{c}=3\underset{\sim}{i}-\underset{\sim}{j}-4\underset{\sim}{k}$ and that $\underset{\sim}{p}=3\underset{\sim}{a}-2\underset{\sim}{b}+\underset{\sim}{c}$, find the magnitude of $\underset{\sim}{p}$ to 1 decimal place.

6. A point P divides **AB** with coordinates $A(2,-1,4)$ and $B(6,-3,5)$ externally in the ratio $3:1$. Find the coordinates of P and the magnitude of **OP**.

7. A point $P(-19,12,-3)$ divides **AB** in the ratio $7:-5$ where A is the point $A(2,5,4)$. Find the coordinates of B and the magnitude of **AB**.

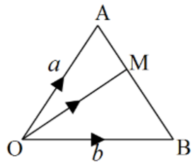
8. The position vector of **A** and **B** are $\underset{\sim}{a}=4\underset{\sim}{i}+4\underset{\sim}{j}-6\underset{\sim}{k}$ and $\underset{\sim}{b}=10\underset{\sim}{i}+4\underset{\sim}{j}+12\underset{\sim}{k}$ respectively. D is a point on AB such that $\mathbf{AD}:\mathbf{DB}=2:1$. Find the midpoint of **OD**.

9. The points P, Q and R lie on a straight line. The position vectors of **P** and **R** are $2\underset{\sim}{i}+3\underset{\sim}{j}+13\underset{\sim}{k}$ and $5\underset{\sim}{i}-3\underset{\sim}{j}+4\underset{\sim}{k}$ respectively. Q divides **PR** externally in the ratio $2:1$. Find the position vector of **Q** and the distance of Q from the origin.

10. The points P, Q and R lie on a straight line. The position vectors of **P** and **R** are $2\underset{\sim}{i}+3\underset{\sim}{j}+3\underset{\sim}{k}$ and $5\underset{\sim}{i}-3\underset{\sim}{j}+4\underset{\sim}{k}$ respectively. Q divides **PR** internally in the ratio $2:1$. Find the position vector of **Q** and the modulus of **OQ**.

11. $\underset{\sim}{a}=\begin{pmatrix}-1\\5\\2\end{pmatrix}$, $\underset{\sim}{b}=\begin{pmatrix}2\\1\\-3\end{pmatrix}$ and $\underset{\sim}{c}=\begin{pmatrix}-8\\7\\13\end{pmatrix}$. Find the scalars s and t such that $s\underset{\sim}{a}+t\underset{\sim}{b}=\underset{\sim}{c}$.

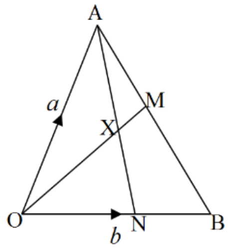
12. In the figure below, $\mathbf{OA}=\underset{\sim}{a}$ and $\mathbf{OB}=\underset{\sim}{b}$. $\mathbf{AM}:\mathbf{MB}=2:3$. Given that $\underset{\sim}{a}=\underset{\sim}{i}-2\underset{\sim}{j}+5\underset{\sim}{k}$ and $\underset{\sim}{b}=\underset{\sim}{i}+3\underset{\sim}{j}-5\underset{\sim}{k}$, calculate the length **OM**.



13. The position vectors of A, B and C are given as $\underset{\sim}{a}=\underset{\sim}{i}+3\underset{\sim}{j}+5\underset{\sim}{k}$, $\underset{\sim}{b}=4\underset{\sim}{i}+12\underset{\sim}{j}+20\underset{\sim}{k}$ and $\underset{\sim}{c}=3\underset{\sim}{i}+9\underset{\sim}{j}+15\underset{\sim}{k}$ respectively. Show that A, B and C are collinear.

14. The position vectors of X and Y are given as $\underset{\sim}{x} = -4\underset{\sim}{i} + a\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{y} = 12\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}$ where a, b and c are integers. Given that the distance of X from the origin is $\sqrt{21}$ and X and Y are parallel, determine:
(a) The possible values of a.
(b) The values of b and c.

15. In the figure below, $\mathbf{OA} = \underset{\sim}{a}$ and $\mathbf{OB} = \underset{\sim}{b}$. The vector $\mathbf{OM} = \frac{2}{3}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b}$ and $\mathbf{AN} = -\underset{\sim}{a} + \frac{3}{5}\underset{\sim}{b}$.

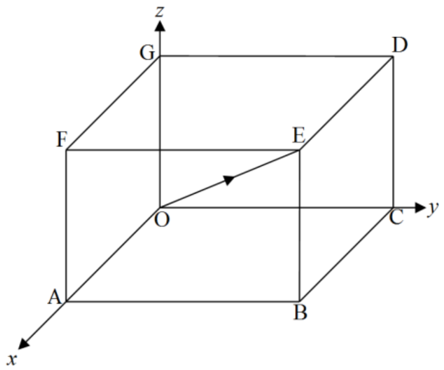


Find the ratio in which X divides:
(i) **OM**
(ii) **AN**

16. The position vectors of A, B and C are given as $\underset{\sim}{a} = x\underset{\sim}{i} - 4\underset{\sim}{j} - 2\underset{\sim}{k}$, $\underset{\sim}{b} = -5\underset{\sim}{i} + y\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c} = 10\underset{\sim}{i} - 8\underset{\sim}{j} + z\underset{\sim}{k}$ respectively, where x, y and z are constants. Given that A, B and C are collinear and that $3AC = -2AB$, find the constants x, y and z.

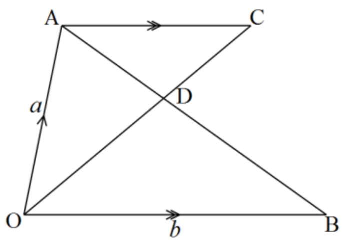
17. Given that $\frac{2}{5}\underset{\sim}{i} + \frac{3}{5}\underset{\sim}{j} + a\underset{\sim}{k}$ is a unit vector, find the value of the constant a.

18. In the figure below, $A = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}$ and $C = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}$. The distance between E and the origin is $5\sqrt{5}$ units. Determine the coordinates of E.



19. The coordinates of A and B are $A(2,5,-1)$ and $B(4,-3,\lambda)$. A point M divides **AB** externally in the ratio $3:2$. If the modulus of **OM** is $5\sqrt{33}$, find the possible values of λ.

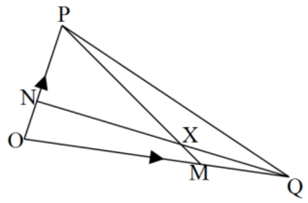
20. In the figure below, $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$. $\overrightarrow{AC} \parallel \overrightarrow{OB}$ and D divides $\overrightarrow{OC}$ in the ratio $5:4$. Find:



(a) $\overrightarrow{AC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$
(b) Sate the ratio in which D divides $\overrightarrow{AB}$.

Section II Questions

1. In the figure below, OPQ is a triangle in which $\mathbf{OP} = \underset{\sim}{p}$ and $\mathbf{OQ} = \underset{\sim}{q}$. M and N are points on OQ and OP respectively such that $\mathbf{ON}:\mathbf{NP} = 1:3$ and $\mathbf{OM}:\mathbf{MQ} = 2:1$.



(a) Express the following vectors in terms of $\underset{\sim}{p}$ and $\underset{\sim}{q}$.

(i) **PQ** (1 mark)

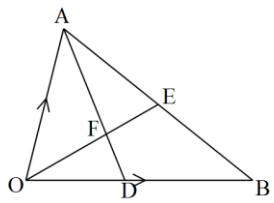
(ii) **PM** (1 mark)

(iii) **QN** (1 mark)

(b) Lines PM and QN intersect at X such that $\mathbf{PX} = h\mathbf{PM}$ and $\mathbf{QX} = k\mathbf{QN}$. Express **OX** in two different ways and find the value of h and k. (6 marks)

(c) OX produced meets PQ at Y such that $PY:YQ = 3:2$. Using the ratio theorem or otherwise, find **OY** in terms of $\underset{\sim}{p}$ and $\underset{\sim}{q}$. (1 mark)

2. In the figure below, E is the mid – point of **AB**. $\mathbf{OD}:\mathbf{DB} = 2:3$ and F is the point of intersection of **OE** and **AD**.



(a) Given that $\mathbf{OA} = \underset{\sim}{a}$ and $\mathbf{OB} = \underset{\sim}{b}$, express in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$:

(i) **OE** (2 marks)

(ii) **AD** (1 mark)

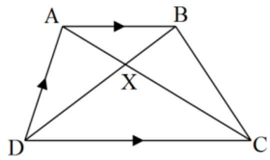
(b) Given further that $\mathbf{AD} = t\mathbf{AF}$ and $\mathbf{OF} = s\mathbf{OE}$;

(i) Express **OF** in two different ways. (2 marks)

(ii) Hence find the values of the scalars s and t. (3 marks)

(c) Show that O, F and E are collinear. (2 marks)

3. In the figure below, ABCD is a trapezium. AB is parallel to DC. Diagonals AC and DB intersect at X and $\mathbf{DC} = 2\mathbf{AB}$, $\mathbf{AB} = \underset{\sim}{a}$, $\mathbf{DA} = \underset{\sim}{d}$, $\mathbf{AX} = k\mathbf{AC}$ and $\mathbf{DX} = h\mathbf{DB}$, where h and k are constants.



(a) Find in terms of $\underset{\sim}{a}$ and $\underset{\sim}{d}$, the vectors:

(i) **BC** (1 mark)

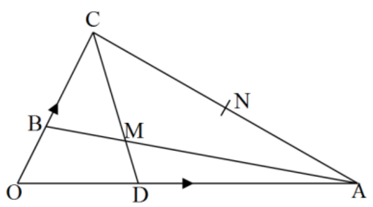
(ii) **AX** (2 marks)

(iii) **DX** (1 mark)

(b) Determine the values of the constants h and k. (5 marks)

(c) Hence state the ratio in which X divides BD. (1 mark)

4. In the figure below, $\overrightarrow{OB} = \underset{\sim}{b}$, $\overrightarrow{OC} = 3\overrightarrow{OB}$ and $\overrightarrow{OA} = \underset{\sim}{a}$. $\overrightarrow{OD} = \frac{1}{3}\overrightarrow{OA}$ and N is the mid – point of $\overrightarrow{AC}$. CD and AB intersect at M.



(a) Determine in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$,

(i) $\overrightarrow{AB}$ (1 mark)

(ii) $\overrightarrow{CD}$ (1 mark)

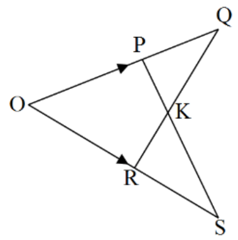
(b) Given that $\overrightarrow{CM} = k\overrightarrow{CD}$ and $\overrightarrow{AM} = h\overrightarrow{AB}$ where h and k are scalars;

(i) Express $\overrightarrow{OM}$ in two different ways. (2 marks)

(ii) Hence find the scalars h and k. (3 marks)

(c) Show that O, M and N are collinear. (3 marks)

5. In the figure below, $\mathbf{OP} = \mathbf{p}$, $\mathbf{OR} = \mathbf{r}$, $\mathbf{OS} = 2\mathbf{r}$ and $\mathbf{OQ} = \frac{3}{2}\mathbf{p}$.



(a) Express the following vectors in terms of $\mathbf{p}$ and $\mathbf{r}$.

(i) **QR** (2 marks)

(ii) **PS** (1 mark)

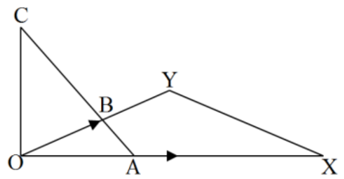
(b) Given also that $\mathbf{QK} = \text{m}\mathbf{QR}$ and $\mathbf{PK} = \text{n}\mathbf{PS}$, express **OK** in two different ways hence find the values of the scalars m and n. (5 marks)

(c) State the ratios:

(i) QK : QR (1 mark)

(ii) PS : SK (1 mark)

6. In the figure below, $\mathbf{OY} = 2\mathbf{OB}$, $\mathbf{OX} = \frac{5}{2}\mathbf{OA}$, $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$.



(a) Express the following in terms of $\mathbf{a}$ and $\mathbf{b}$.

(i) **AB** (1 mark)

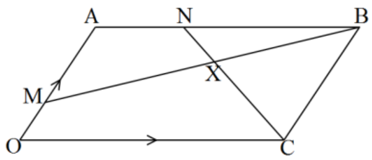
(ii) **XY** (1 mark)

(b) Given that $\mathbf{AC} = 6\mathbf{AB}$, express **OC** and **XC** in terms of $\mathbf{a}$ and $\mathbf{b}$. (4 marks)

(c) Show that X, Y and C are collinear. (2 marks)

(d) State the ratio in which C divides XY. (2 marks)

7. The figure below is a parallelogram OABC in which $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$. $\overrightarrow{OM} = \frac{1}{3}\overrightarrow{OA}$, $\overrightarrow{AN} = \frac{2}{5}\overrightarrow{AB}$ and BM intersect with CN at X.



(a) Find the following vectors in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

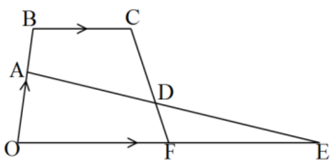
(i) $\overrightarrow{BM}$ (1 mark)

(ii) $\overrightarrow{CN}$ (1 mark)

(b) Given that $\overrightarrow{BM} = k\overrightarrow{BX}$ and $\overrightarrow{CN} = h\overrightarrow{CX}$ where h and k are constants, express $\overrightarrow{CX}$ in two ways hence find the constants h and k. (5 marks)

(c) Given further that $\underset{\sim}{a} = 4\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{c} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 5\underset{\sim}{k}$, find the length of $\overrightarrow{CX}$. (3 marks)

8. In the figure below, $\mathbf{OA} = \underset{\sim}{x}$ and $\mathbf{BC} = \underset{\sim}{y}$. $\mathbf{OE} = 3\mathbf{BC}$, $\mathbf{OA} : \mathbf{AB} = 2:1$ and F is the midpoint of **OE**.



(a) Find in terms of $\underset{\sim}{x}$ and $\underset{\sim}{y}$, the following vectors.

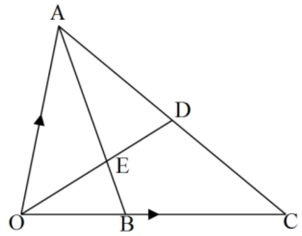
(i) **FC** (2 marks)

(ii) **AE** (1 mark)

(b) Given that $\mathbf{FD} = \text{m}\mathbf{FC}$ and $\mathbf{AD} = \text{n}\mathbf{AE}$ where m and n are scalars, express **FD** in two different ways hence find the values of the scalars m and n. (5 marks)

(c) Show that A, D and E are collinear. (2 marks)

9. In the figure below, $\mathbf{OA} = \underset{\sim}{a}$ and $\mathbf{OB} = \underset{\sim}{b}$. $\mathbf{OB} = \frac{2}{5}\mathbf{OC}$, D is the midpoint of **AC** and **AB** intersect **OD** at E.



(a) Express the following vectors in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ only:

(i) **AB** (1 mark)

(ii) **OD** (1 mark)

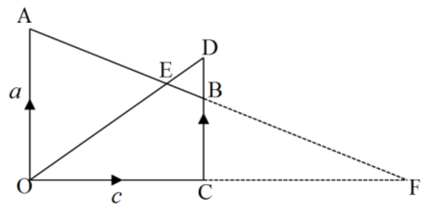
(b) Given further that $\mathbf{AE} = r\mathbf{AB}$ and $\mathbf{OE} = s\mathbf{OD}$ where r and s are parameters, express **OE** in two different ways hence find the values of r and s. (5 marks)

(c) State the ratio in which E divides:

(i) **AB** (1 mark)

(ii) **OD** (1 mark)

10. In the figure below, $\mathbf{OA} = \underset{\sim}{a}$ and $\mathbf{OC} = \underset{\sim}{c}$. $\mathbf{CB} = \frac{2}{3}\mathbf{OA}$ and B divides CD in the ratio $3:1$.



(a) Express the following vectors in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$ only:

(i) **AB** (1 mark)

(ii) **OD** (2 marks)

(b) Given that $\mathbf{OE} = h\mathbf{OD}$ and $\mathbf{AE} = k\mathbf{AB}$ where h and k are scalars express **OE** in two different ways hence find the scalars h and k. (5 marks)

(c) If **OC** produced meets **AB** produced at F, find **OF**. (3 marks)

Section I Answers

1. $a = \pm\frac{2}{3}$

2. $\left|\underset{\sim}{p}\right| = 3.5$, $\underset{\sim}{q} = 6\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$

3. $\sqrt{3} = 1.7321$

4. $3\sqrt{43} = 19.7$

5. $\sqrt{318} \simeq 17.8$

6. 10.5

7. $2\sqrt{11} = 6.633$

8. $(4,2,3)$

9. $\underset{\sim}{q} = 8\underset{\sim}{i} - 9\underset{\sim}{j} - 5\underset{\sim}{k}$, $|\mathbf{OQ}| = 13.04$

10. $\underset{\sim}{q} = 4\underset{\sim}{i} - \underset{\sim}{j} + 3\frac{2}{3}\underset{\sim}{k}$, 5.5176

11. $s = 2,\ t = -3$

12. $\sqrt{2} = 1.414$

13. $\frac{3}{2}\overrightarrow{AC} = \overrightarrow{AB}$ hence $\overrightarrow{AC} \parallel \overrightarrow{AB}$
A is common
$\therefore$ A, B and C are collinear

14. $a = 2,\ b = -6,\ c = -3$
$a = -2,\ b = 6,\ c = -3$

15. (a) $9:2$ (b) $5:6$

16. $x = 4,\ y = 2,\ z = -4$

17. $a = \pm\dfrac{2\sqrt{3}}{5}$

18. $E(5,6,8)$

19. $\lambda = 6,\ \lambda = -7\frac{1}{3}$

20. (a) $\overrightarrow{AC} = \frac{4}{5}\underset{\sim}{b}$ (b) $\overrightarrow{AD} : \overrightarrow{DB} = 4:5$

Section I Answers

1. (a) (i) $\mathbf{PQ} = \underset{\sim}{q} - \underset{\sim}{p}$ (ii) $\mathbf{PM} = \frac{2}{3}\underset{\sim}{q} - \underset{\sim}{p}$ (iii) $\mathbf{QN} = -\underset{\sim}{q} + \frac{1}{4}\underset{\sim}{p}$ (b) $h = \frac{9}{10}$, $k = \frac{2}{5}$ (c) $OY = \frac{2}{5}\underset{\sim}{p} + \frac{3}{5}\underset{\sim}{q}$

2. (a) (i) $\mathbf{OE} = \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$ (ii) $\mathbf{AD} = -\underset{\sim}{a} + \frac{2}{5}\underset{\sim}{b}$ (b) (i) $\mathbf{OF} = \frac{1}{2}s\underset{\sim}{a} + \frac{1}{2}s\underset{\sim}{b}$, $\mathbf{OF} = \underset{\sim}{a} - \frac{1}{t}\underset{\sim}{a} + \frac{2}{5t}\underset{\sim}{b}$ (ii) $s = \frac{4}{7}$, $t = 1\frac{2}{5}$
(c) $\mathbf{OF} = \frac{4}{7}\mathbf{OE}$ hence $\mathbf{OF} \parallel \mathbf{OE}$, O is common, $\therefore$ O, F and E are collinear

3. (a) (i) $\mathbf{BC} = \underset{\sim}{a} - \underset{\sim}{d}$ (ii) $\mathbf{AX} = 2k\underset{\sim}{a} - k\underset{\sim}{d}$ (iii) $\mathbf{DX} = h\underset{\sim}{a} + h\underset{\sim}{d}$ (b) $h = \frac{2}{3}$ and $k = \frac{1}{3}$ (c) $BX : XD = 1:2$

4. (a) (i) $\mathbf{AB} = \underset{\sim}{b} - \underset{\sim}{a}$ (ii) $\mathbf{CD} = \frac{1}{3}\underset{\sim}{a} - 3\underset{\sim}{b}$ (b) (i) $\mathbf{OM} = 3\underset{\sim}{b} + \frac{1}{3}k\underset{\sim}{a} - 3k\underset{\sim}{b}$, $\mathbf{OM} = \underset{\sim}{a} + h\underset{\sim}{b} - h\underset{\sim}{a}$ (ii) $h = k = \frac{3}{4}$
(c) $2\mathbf{OM} = \mathbf{ON}$ hence $\mathbf{OM} \parallel \mathbf{ON}$, O is common, $\therefore$ O, M and N are collinear

5. (a) (i) $\mathbf{QR} = -\frac{3}{2}\underset{\sim}{p} + \underset{\sim}{r}$ (ii) $\mathbf{PS} = -\underset{\sim}{p} + 2\underset{\sim}{r}$ (b) $\mathbf{OK} = \underset{\sim}{p} - n\underset{\sim}{p} + 2n\underset{\sim}{r}$, $\mathbf{OK} = \frac{3}{2}\underset{\sim}{p} - \frac{3}{2}m\underset{\sim}{p} + m\underset{\sim}{r}$, $m = \frac{1}{2}$, $n = \frac{1}{4}$
(c) (i) $QK : QR = 1:2$ (ii) $PS : SK = 4:-3$

6. (a) (i) $\mathbf{AB} = \underset{\sim}{b} - \underset{\sim}{a}$ (ii) $\mathbf{XY} = 2\underset{\sim}{b} - \frac{5}{2}\underset{\sim}{a}$ (b) $\mathbf{OC} = 6\underset{\sim}{b} - 5\underset{\sim}{a}$, $\mathbf{XC} = 6\underset{\sim}{b} - \frac{15}{2}\underset{\sim}{a}$
(c) $2\mathbf{XY} = \mathbf{XC}$ hence $\mathbf{XY} \parallel \mathbf{XC}$, X is common, $\therefore$ X, Y and C are collinear

7. (a) (i) $\mathbf{BM} = -\frac{2}{3}\underset{\sim}{a} - \underset{\sim}{c}$ (ii) $\mathbf{CN} = \underset{\sim}{a} - \frac{3}{5}\underset{\sim}{c}$ (b) $h = 1\frac{2}{5}$, $k = 2\frac{1}{3}$ (c) $|\mathbf{CX}| = 4.4193$

8. (a) (i) $\mathbf{FC} = \frac{3}{2}\underset{\sim}{x} - \frac{1}{2}\underset{\sim}{y}$ (ii) $\mathbf{AE} = -\underset{\sim}{x} + 3\underset{\sim}{y}$ (b) $m = \frac{3}{8}$, $n = \frac{7}{16}$
(c) $\frac{7}{16}\mathbf{AE} = \mathbf{AD}$ hence $\mathbf{AE} \parallel \mathbf{AD}$, A is common, $\therefore$ A, E and D are collinear

9. (a) (i) $\mathbf{AB} = \underset{\sim}{b} - \underset{\sim}{a}$ (ii) $\mathbf{OD} = \frac{1}{2}\underset{\sim}{a} + \frac{5}{4}\underset{\sim}{b}$ (b) $r = \frac{5}{7}$, $s = \frac{4}{7}$ (c) (i) $5:2$ (ii) $4:3$

10. (a) (i) $\mathbf{AB} = \underset{\sim}{c} - \frac{1}{3}\underset{\sim}{a}$ (ii) $\mathbf{OD} = \frac{8}{9}\underset{\sim}{a} + \underset{\sim}{c}$ (b) $h = k = \frac{9}{11}$ (c) $\mathbf{OF} = 3\underset{\sim}{c}$