

# CALCULUS.

## Mixed Exercise.

1. Find  $\frac{dy}{dx}$  if  $y = (x^2 + 1)^2$ .

$$y = (x^2 + 1)^2$$

$$y = (x^2 + 1)(x^2 + 1)$$

$$y = (x^2)^2 + 2(x^2)(1) + (1)^2$$

$$y = x^4 + 2x^2 + 1$$

$$\frac{dy}{dx} = 4x^3 + 4x.$$

2. Find the equation of the tangent to the curve  $y = 6x^2 - x - 4$  at the point A(1,1).

Curve;  $y = 6x^2 - x - 4$  at A(1, 1)

Gradient of tangent;

$$\frac{dy}{dx} = 12x - 1$$

$$\text{at } x = 1$$

$$M_1 = 12(1) - 1$$

$$= 11$$

For the tangent  $M_1 = M_2$

$$= 11$$

By taking Point A(1, 1)

$$M_2 = 11$$

and Point (x, y)

$$\frac{y - 1}{x - 1} = 11$$

$$y = 11x - 11 + 1$$

$$y = 11x - 10.$$

3. Find the equation of the tangent to the curve  $y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + x - 2$  at a point where  $x = 1$ . Give your answer in the form  $ax + by + c = 0$  where  $a, b$  and  $c$  are integers.

**Curve;  $y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + x - 2$**

**at  $x = 1$ .**

**Gradient of tangent;**

$$\frac{dy}{dx} = x^2 - 3x + 1$$

**at  $x = 1$**

$$M_1 = (1)^2 - 3(1) + 1$$

$$= -1$$

$$y = \frac{1}{3}(1)^3 - \frac{3}{2}(1)^2 + (1) - 2$$

$$y = -\frac{13}{6}$$

**Point of tanging  $(1, -\frac{13}{6})$**

**Equation;**

$$\frac{y + \frac{13}{6}}{x - 1} = -1$$

$$y = -x + 1 - \frac{13}{6}$$

$$6y = -6x - 7$$

$$6x + 6y + 7 = 0.$$

4. The gradient of the tangent to the curve  $y = ax^3 + bx^2$  at the point  $A(-1, -5)$  is 12. Find the value  $a$  and  $b$ .

$$y = ax^3 + bx^2$$

**at  $A(-1, -5)$**

**$x = -1$  and  $y = -5$**

**$-a + b = -5 \dots \dots \text{eq. 1}$**

$$\frac{dy}{dx} = 3ax^2 + 2bx = 12$$

**at  $x = -1$**

**$3a - 2b = 12 \dots \dots \dots \text{eq. 2}$**

**From eq. 1**

$$b = a - 5$$

**substituting in eq. 2,**

**we have;**

$$3a - 2(a - 5) = 12$$

$$3a - 2a + 10 = 12$$

$$a = 12 - 10$$

$$a = 2$$

$$b = 2 - 5$$

$$b = -3$$

$$a = 2$$

$$b = -3$$

5. Find the equation of the normal to the curve  $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 3x + 8$  at its  $y$  – intercept.

$$y = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 3x + 8$$

at  $y$  – intercept,  $x = 0$

$$y = \frac{1}{3}(0)^3 - \frac{1}{2}(0)^2 + 3(0) + 8$$

$$y = 8$$

Point of tanging  $A(0, 8)$

Gradient of the tangent;

$$\frac{dy}{dx} = x^2 - x + 3$$

$$\text{at } x = 0$$

$$M_1 = (0)^2 - (0) + 3$$

$$M_1 = 3$$

For normal;

$$M_1 M_2 = -1$$

$$M_2 = -\frac{1}{3}$$

Taking Point;

$$A(0, 8), (x, y) \text{ and } M_2 = -\frac{1}{3}$$

$$\frac{y - 8}{x - 0} = -\frac{1}{3}$$

$$y = -\frac{1}{3}x + 8$$

$$3y = -x + 24.$$

6. The gradient of the of the tangent to the curve  $y = ax^3 + \frac{1}{2}x^2 - 3x + 7$  at a point where  $x = 2$  is 5. Find the value of  $a$ .

$$y = ax^3 + \frac{1}{2}x^2 - 3x + 7$$

$$\text{at } x = 2, \frac{dy}{dx} = 5$$

$$\frac{dy}{dx} = 3ax^2 + x - 3$$

$$\text{at } x = 2;$$

$$3a(2)^2 + (2) - 3 = 5$$

$$12a - 1 = 5$$

$$12a = 6$$

$$a = \frac{1}{2}.$$

7. Find the equation of the normal to the curve  $y = \frac{x(9x^2-1)}{3x+1}$  at a point where  $x = 2$ .

$y = \frac{x(9x^2 - 1)}{3x + 1}$	<b>Point of tanging;</b>	<b>Gradient of normal;</b>
$y = \frac{x(3x - 1)(3x + 1)}{(3x + 1)}$	A(2, 10)	$M_1 M_2 = -1$
$y = x(3x - 1)$	<b>Gradient of Tangent;</b>	$M_2 = -\frac{1}{11}$
$y = 3x^2 - x$	$\frac{dy}{dx} = 6x - 1$	<b>Equation;</b>
at $x = 2$	at $x = 2$	$\frac{y - 10}{x - 2} = -\frac{1}{11}$
$y = 3(2)^2 - (2)$	$M_1 = 6(2) - 1$	$11y - 110 = -x + 2$
$y = 10$	$= 11$	$11y + x - 112 = 0.$

8. Determine the point on the curve  $y = \frac{1}{2}x^2 + 4$  at which the gradient is 8 hence find the equation of the normal to the curve at this point.

$y = \frac{1}{2}x^2 + 4$	<b>Gradient of normal;</b>
$\frac{dy}{dx} = x$	$M_1 M_2 = -1$
$x = 8$	$M_2 = -\frac{1}{8}$
$y = \frac{1}{2}(8)^2 + 4$	<b>Equation;</b>
$= 36$	$\frac{y - 36}{x - 8} = -\frac{1}{8}$
<b>Point of tanging;</b>	$8y - 288 = -x + 8$
A(8, 36)	$8y + x - 296 = 0.$

9. Find the equation of the tangent to the curve  $y = x^2 - 3x + 7$  at a point where the gradient is 5.

$$y = x^2 - 3x + 7$$

$$\frac{dy}{dx} = 2x - 3$$

$$= 5$$

$$2x - 3 = 5$$

$$2x = 8$$

$$x = 4$$

$$y = (4)^2 - 3(4) + 7$$

$$= 11$$

**Point of tanging;**

$$A(4, 11)$$

**Equation;**

$$\frac{y - 11}{x - 4} = 5$$

$$y = 5x - 20 + 11$$

$$y = 5x - 9.$$

10. Find the equation of the tangents to the curve  $y = 5x - x^2$  at the points where it intersect with the line  $y = x$ .

$$y = 5x - x^2 \dots \dots \text{eq. 1}$$

$$y = x \dots \dots \text{eq. 2}$$

**Substituting eq. 2 into eq. 1**

$$5x - x^2 = x$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$x = 0$$

or

$$x - 4 = 0$$

$$x = 4$$

when  $x = 0$ ;

$$y = 5(0) - (0)^2$$

$$y = 0$$

**Point of tanging;**

$$A(0, 0)$$

**Gradient of the tangent;**

$$\frac{dy}{dx} = 5 - 2x$$

$$\text{at } x = 0$$

$$\frac{dy}{dx} = 5 - 2(0)$$

$$= 5$$

**Equation;**

$$\frac{y - 0}{x - 0} = 5$$

$$y = 5x \dots \dots \text{1st equation.}$$

or;

$$\text{at } x = 4$$

$$y = 5(4) - (4)^2$$

$$= 4$$

**Point of tanging;**

$$B(4, 4)$$

**Gradient of the tangent;**

$$\frac{dy}{dx} = 5 - 2x$$

$$\text{at } x = 4$$

$$\frac{dy}{dx} = 5 - 2(4)$$

$$= -3$$

**Equation;**

$$\frac{y - 4}{x - 4} = -3$$

$$y = -3x + 12 + 4$$

$$y = -3x + 16 \dots \dots \text{2nd equation.}$$

11. Find the coordinates of the stationary points of the curve  $y = -x^3 + 3x^2 + 9x - 6$ .

$$y = -x^3 + 3x^2 + 9x - 6$$

At stationary point;

$$\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -3x^2 + 6x + 9$$

$$-3x^2 + 6x + 9 = 0$$

$$x = \frac{-6 \pm \sqrt{(-6)^2 - 4(-3)(9)}}{2(-3)}$$

$$x = \frac{-6 \pm 12}{-6}$$

$$x = -1 \text{ or } x = 3$$

when  $x = -1$

$$y = -(-1)^3 + 3(-1)^2 + 9(-1) - 6 \\ = -11$$

Turning point  $(-1, -11)$

when  $x = 3$

$$y = -(3)^3 + 3(3)^2 + 9(3) - 6 \\ = 21$$

Turning point  $(3, 21)$ .

12. Find the coordinates of the stationary points of the curve  $y = 2x^3 - 3x^2 - 36x - 8$ .

$$y = 2x^3 - 3x^2 - 36x - 8$$

At stationary point  $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 6x^2 - 6x - 36$$

$$6x^2 - 6x - 36 = 0$$

$$x^2 - x - 6 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{1 \pm 5}{2}$$

$$x = 3 \text{ or } x = -2$$

when  $x = 3$

$$y = 6(3)^2 - 6(3) - 36 \\ = -89$$

Turning point  $(3, -89)$

when  $x = -2$

$$y = 6(-2)^2 - 6(-2) - 36 \\ = 36$$

Turning point  $(-2, 36)$

13. Given that  $y = x^3 - 4x$ , find;  
 (a) The intercepts of the curve.

$y = x^3 - 4x$	<b>at x – intercept, <math>y = 0</math></b>	$x^2 = 4$
<b>at y – intercept, <math>x = 0</math></b>	$x^3 - 4x = 0$	$x = \sqrt{4}$
$y = (0)^3 - 4(0)$	$x(x^2 - 4) = 0$	$x = \pm 2$
$y = 0$	$x = 0$	<b>Coordinates <math>(-2, 0)</math> or <math>(2, 0)</math></b>
<b>Coordinates <math>(0, 0)</math></b>	$x^2 - 4 = 0$	

- (b) The stationary points of the curve.

<b>at stationary point <math>\frac{dy}{dx} = 0</math></b>	$y = \left(\frac{2}{\sqrt{3}}\right)^3 - 4\left(\frac{2}{\sqrt{3}}\right)$
$\frac{dy}{dx} = 3x^2 - 4$	$= \frac{8}{3\sqrt{3}} - 4\left(\frac{2}{\sqrt{3}}\right)$
$3x^2 - 4 = 0$	$= 1.5396$
$3x^2 = 4$	<b>Turning point <math>(1.1547, 1.5396)</math></b>
$x^2 = \frac{4}{3}$	<b>when <math>x = -\frac{2}{\sqrt{3}}</math></b>
$x = \sqrt{\frac{4}{3}}$	$y = \left(-\frac{2}{\sqrt{3}}\right)^3 - 4\left(-\frac{2}{\sqrt{3}}\right)$
$x = \pm \frac{2}{\sqrt{3}}$	$= -\frac{8}{3\sqrt{3}} + 4\left(\frac{2}{\sqrt{3}}\right)$
<b>when <math>x = \frac{2}{\sqrt{3}}</math></b>	$= 3.0792$
	<b>Turning point <math>(-1.1547, 3.0792)</math></b>

14. Investigate the stationary points for the curve  $y = 2x^3 + 5$ .

$$y = 2x^3 + 5$$

At stationary point;

$$\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 6x^2$$

$$6x^2 = 0$$

$$x^2 = 0$$

$$x = \pm 0$$

At  $x = 0$ ,

$$y = 2(0)^3 + 5$$

$$y = 0$$

Coordinates (0, 5)

Nature of turning point;

<b>x</b>	<b>-1</b>	<b>0</b>	<b>1</b>
$\frac{dy}{dx} = 6x^2$	<b>6</b>	<b>0</b>	<b>6</b>
<b>Nature</b>			

Point (0, 5) is a point of inflection

15. Identify the stationary points of the curve  $y = 2x^2 - x^4 + 2$ .

$$y = 2x^2 - x^4 + 2$$

At stationary point;

$$\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 4x - 4x^3$$

$$4x - 4x^3 = 0$$

$$4x(1 - x^2) = 0$$

$$4x(1 - x)(1 + x) = 0$$

Either  $x = 0$  or  $x = -1$  or  $x = 1$

at  $x = 0$

$$y = 2(0)^2 - (0)^4 + 2$$

$$y = 2$$

Turning point (0, 2)

At  $x = -1$

$$y = 2(-1)^2 - (-1)^4 + 2$$

$$= 3$$

Turning point (-1, 3)

At  $x = 1$

$$y = 2(1)^2 - (1)^4 + 2$$

$$= 3$$

Turning point (1, 3).

16. The gradient of a curve is given by  $\frac{dy}{dx} = 3x^2 - 8x + 2$ . If the curve passes through a point (2,5), find its equation.

$$\frac{dy}{dx} = 3x^2 - 8x + 2$$

$$y = \int (3x^2 - 8x + 2) dx$$

$$y = x^3 - 4x^2 + 2x + c$$

At (2, 5)

$$5 = (2)^3 - 4(2)^2 + 2(2) + c$$

$$c = 5 + 4$$

$$c = 9$$

$$y = x^3 - 4x^2 + 2x + 9.$$

17. The gradient of a curve is given by  $\frac{dy}{dx} = 4x - 2$ . If the minimum value of the curve is 7, find its equation.

$$\frac{dy}{dx} = 4x - 2$$

$$y = \int (4x - 2) dx$$

$$y = 2x^2 - 2x + c$$

At minimum point  $\frac{dy}{dx} = 0$

$$4x - 2 = 0$$

$$4x = 2$$

$$x = \frac{1}{2}$$

$$\text{at } x = \frac{1}{2}, y = 7$$

$$7 = 2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + c$$

$$7 = -\frac{1}{2} + c$$

$$c = 7\frac{1}{2}$$

$$y = 2x^2 - 2x + 7\frac{1}{2}.$$

18. Evaluate  $\int_{t=2}^4 (3t^2 + 4t + 10)dt$ .

$$\begin{aligned} & \int_{t=2}^4 (3t^2 + 4t + 10)dt \\ &= [t^3 + 2t^2 + 10t]_2^4 \\ &= [(4)^3 + 2(4)^2 + 10(4)] - [(2)^3 + 2(2)^2 + 10(2)] \end{aligned}$$

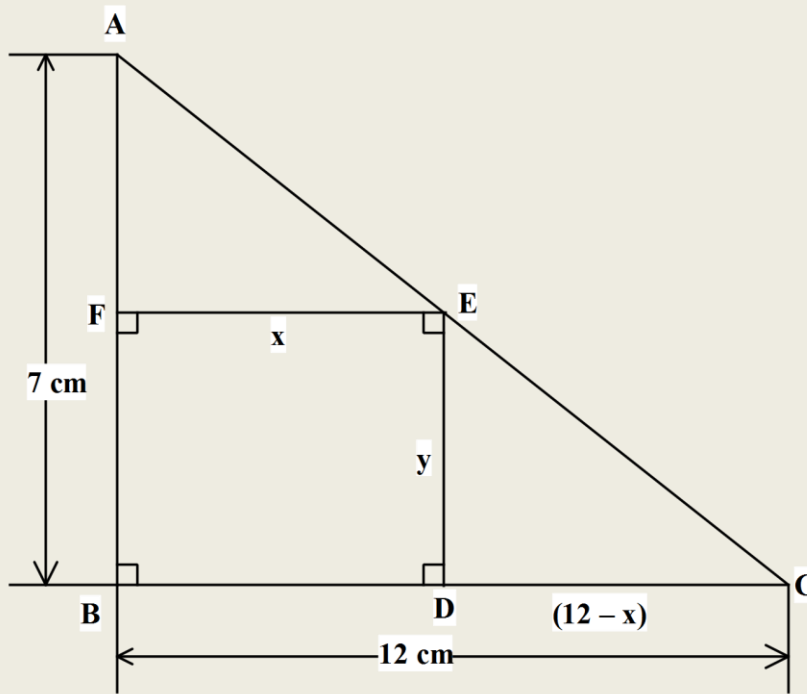
$$\begin{aligned} &= 136 - 36 \\ &= 100 \text{ square units.} \end{aligned}$$

19. Find the value of a if  $\int_a^3 (2x + 4) dx = 25$ .

$$\begin{aligned} & \int_a^3 (2x + 4) dx = 25 \\ & [x^2 + 4x]_a^3 = 25 \\ & [(3)^2 + 4(3)] - [(a)^2 + 4(a)] = 25 \\ & 21 - a^2 - 4a = 25 \end{aligned}$$

$$\begin{aligned} & a^2 + 4a + 4 = 0 \\ & a = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(4)}}{2(1)} \\ & a = \frac{-4 \pm 0}{2} \\ & = -2. \end{aligned}$$

20. A rectangle with maximum possible area is inscribed in a right angle triangle with base 12 cm and height 7 cm. Determine the dimensions of the rectangle.



Consider  $\triangle ABC$  and  $\triangle EDC$

$$\frac{AB}{ED} = \frac{BC}{DC}$$

$$\frac{7}{y} = \frac{12}{(12 - x)}$$

$$12y = 84 - 7x$$

$$y = 7 - \frac{7}{12}x \dots \dots \text{eq. 1}$$

But area of rec. FEDB

$$A = xy$$

$$A = x \left( 7 - \frac{7}{12}x \right)$$

$$A = 7x - \frac{7}{12}x^2.$$

At max. area  $\frac{dA}{dx} = 0$

$$7 - \frac{7}{6}x = 0$$

$$\frac{7}{6}x = 7$$

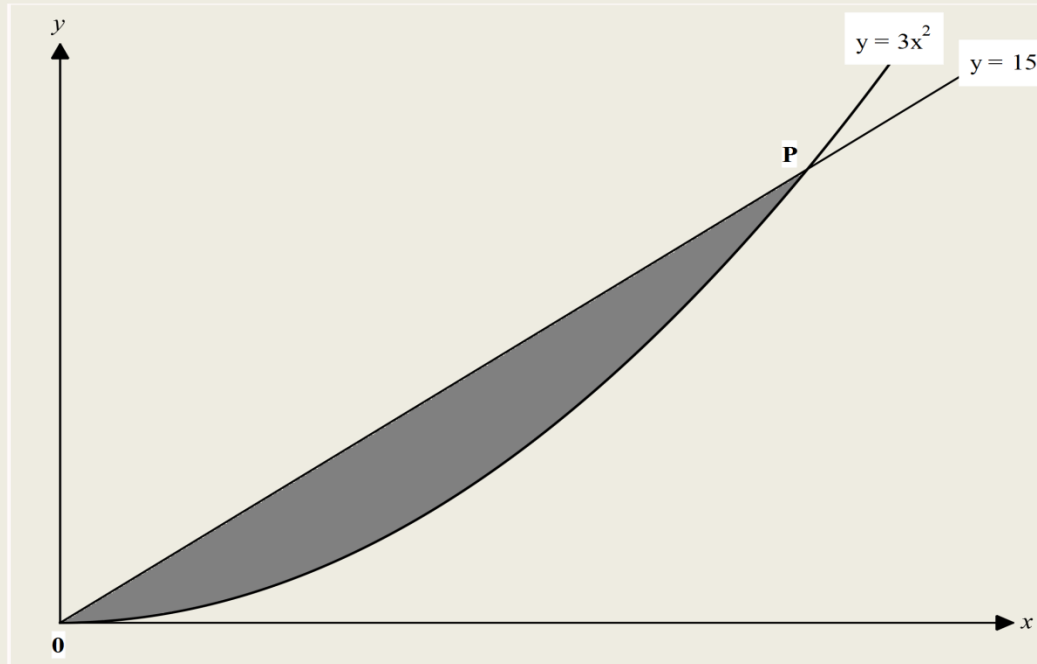
$$x = 6 \text{ cm}$$

$$y = 7 - \frac{7}{12}(6)$$

$$y = 3.5 \text{ cm.}$$

# Area Under a Curve.

1. In the figure below, the shaded region is bounded between the line  $y = 15x$  and the curve  $y = 3x^2$ .



(a) Determine the coordinates of P.

(3mks)

**At 0 and 0;**

$$3x^2 = 15x \text{ (Point of intersection)}$$

$$3x^2 - 15x = 0$$

$$3x(x - 5) = 0$$

$$3x = 0$$

$$x = 0$$

$$x - 5 = 0$$

$$x = 5$$

**when  $x = 5$ ;**

$$y = 15(5)$$

$$= 75$$

$$\therefore P(5, 75).$$

(b) By integration, determine the area of the shaded region.

(3mks)

**Area of the shaded;**

$$= \int_{x=0}^5 (15x) dx - \int_{x=0}^5 (3x^2) dx$$
$$= \left[ \frac{15}{2} x^2 \right]_0^5 - [x^3]_0^5$$

$$= \left[ \left( \frac{15}{2} (5)^2 \right) - \left( \frac{15}{2} (0)^2 \right) \right] - [5^3 - 0^3]$$
$$= 187.5 - 125$$
$$= 62.5 \text{ sq. units.}$$

(c) Estimate the area of the shaded region using trapezoidal rule with 5 strips.

(4mks)

**Area under  $y = 15x$ ;**

$$h = \frac{5 - 0}{5}$$
$$= 1 \text{ unit.}$$

x	0	1	2	3	4	5
y	0	15	30	45	60	75

**Area;**

$$= \frac{1}{2} [(0 + 75) + 2(15 + 30 + 45 + 60)]$$
$$= \frac{1}{2} [75 + 300]$$
$$= 187.5 \text{ square units}$$

**Area under  $y = 3x^2$ .**

x	0	1	2	3	4	5
y	0	3	12	27	45	75

**Area;**

$$= \frac{1}{2} [(0 + 75) + 2(3 + 12 + 27 + 45)]$$
$$= \frac{1}{2} [75 + 180]$$
$$= 127.5 \text{ sq. units}$$

**Area of the shaded region;**

$$= 187.5 - 127.5$$
$$= 60 \text{ sq. units.}$$

2. (a) complete the table below for the function  $y = x^2 - 3x + 6$  in the range  $-2 \leq x \leq 8$ . (2mks)

<b>x</b>	<b>-2</b>	<b>-1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>
<b>y</b>	<b>16</b>	<b>10</b>	<b>6</b>	<b>4</b>	<b>4</b>	<b>6</b>	<b>10</b>	<b>16</b>	<b>24</b>	<b>34</b>	<b>46</b>

- (b) Use the trapezium rule with 5 strips to estimate the area bounded by the curve  $y = x^2 - 3x + 6$  in the range  $-2 \leq x \leq 8$  and the  $x$  - axis. (3mks)

$$h = \frac{8 - (-2)}{5}$$

$$= 2$$

$$A = \frac{2}{2} \{ (16 + 46) - 2(6 + 4 + 10 + 24) \}$$

$$= 1 \{ 62 + 88 \}$$

$$= 150 \text{ sq. units.}$$

- (c) Use the mid-ordinate with 5 strips to estimate the area bounded by the curve  $y = x^2 - 3x + 6$  in the range  $-2 \leq x \leq 8$  and the  $x$  - axis. (2mks)

$$h = \frac{8 - (-2)}{5}$$

$$= 2$$

$$A = 2(10 + 4 + 6 + 6 + 16 + 34)$$

$$= 140 \text{ sq. units.}$$

- (d) By integration, determine the actual area bounded by the curve  $y = x^2 - 3x + 6$  in the range  $-2 \leq x \leq 8$  and the  $x$  - axis. (3mks)

$$A = \int_{x=-2}^8 (x^2 - 3x + 6) dx$$

$$= \left[ \frac{1}{3}x^3 - \frac{3}{2}x^2 + 6x \right]_{-2}^8$$

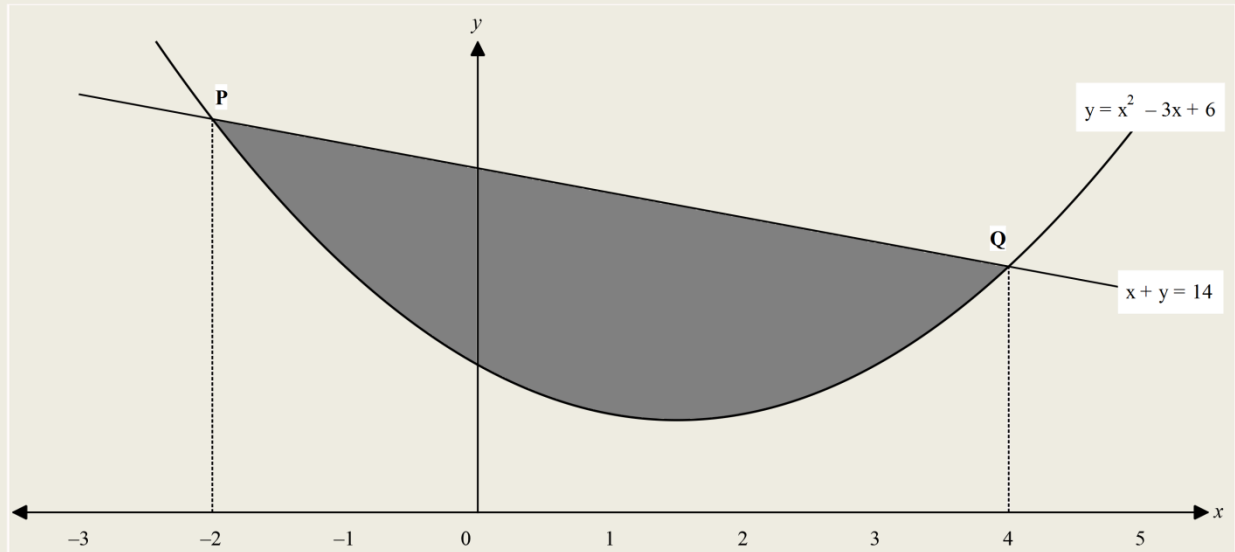
$$= \left[ \frac{1}{3}(8)^3 - \frac{3}{2}(8)^2 + 6(8) \right] - \left[ \frac{1}{3}(-2)^3 - \frac{3}{2}(-2)^2 + 6(-2) \right]$$

$$= \left[ \frac{368}{3} \right] - \left[ -\frac{62}{3} \right]$$

$$= \frac{430}{3}$$

$$= 143 \frac{1}{3} \text{ sq. units.}$$

3. The diagram shows a sketch of a curve  $y = x^2 - 3x + 6$  intersecting with the line  $x + y = 14$  for  $-2 \leq x \leq 4$  at points P and Q.



- (a) Find the coordinates of points P and Q.

(1mk)

**At points of intersection;**

$$x^2 - 3x + 6 = -x + 14$$

**At P,  $x = -2$**

$$y = -(-2) + 14$$

$$= 16$$

$$\mathbf{P(-2, 16)}$$

**At Q,**

$$\mathbf{X = 4}$$

$$\mathbf{Y = -4 + 14}$$

$$= 10$$

$$\mathbf{Q(4, 10).}$$

- (b) Fill the table below for the values of  $y$  for  $y = x^2 - 3x + 6$ .

(2mks)

<b>x</b>	<b>-2</b>	<b>-1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>y</b>	<b>16</b>	<b>10</b>	<b>6</b>	<b>4</b>	<b>4</b>	<b>6</b>	<b>10</b>

- (c) Determine the area bounded by the curve  $y = x^2 - 3x + 6$  and the line  $x + y = 14$ , using trapezium rule with 6 strips. (4mks)

**area of  $y = x^2 - 3x + 6$**

x	-2	-1	0	1	2	3	4
y	16	10	6	4	4	6	10

$$h = \frac{4 - (-2)}{6}$$

$$= 1$$

$$A = \frac{1}{2} [(16 + 10) + 2(10 + 6 + 4 + 4 + 6)]$$

$$= \frac{1}{2} (26 + 60)$$

$$= 43 \text{ sq. unit}$$

**Area of  $y = -x + 14$ .**

x	-2	-1	0	1	2	3	4
y	16	15	14	13	12	11	10

$$A = \frac{1}{2} [(16 + 10) + 2(15 + 14 + 13 + 12 + 11)]$$

$$= \frac{1}{2} (26 + 130)$$

$$= 78 \text{ sq. unit}$$

**Shaded area;**

$$= 78 - 43$$

$$= 35 \text{ sq. units.}$$

- (d) Calculate the exact area of the shaded region. (3mks)

**Area under  $y = 14 - x$**

$$A = \int_{x=-2}^4 (14 - x) dx$$

$$= \left[ 14x - \frac{1}{2}x^2 \right]_{-2}^4$$

$$= \left[ 14(4) - \frac{1}{2}(4)^2 \right] - \left[ 14(-2) - \frac{1}{2}(-2)^2 \right]$$

$$= 48 - (-30)$$

$$= 78 \text{ sq. units}$$

**Area under  $y = x^2 - 3x + 6$**

$$A = \int_{x=-2}^4 (x^2 - 3x + 6) dx$$

$$= \left[ \frac{1}{3}x^3 - \frac{3}{2}x^2 + 6x \right]_{-2}^4$$

$$= \left[ \frac{1}{3}(4)^3 - \frac{3}{2}(4)^2 + 6(4) \right] - \left[ \frac{1}{3}(-2)^3 - \frac{3}{2}(-2)^2 + 6(-2) \right]$$

$$= \frac{64}{3} - \left[ -\frac{62}{3} \right]$$

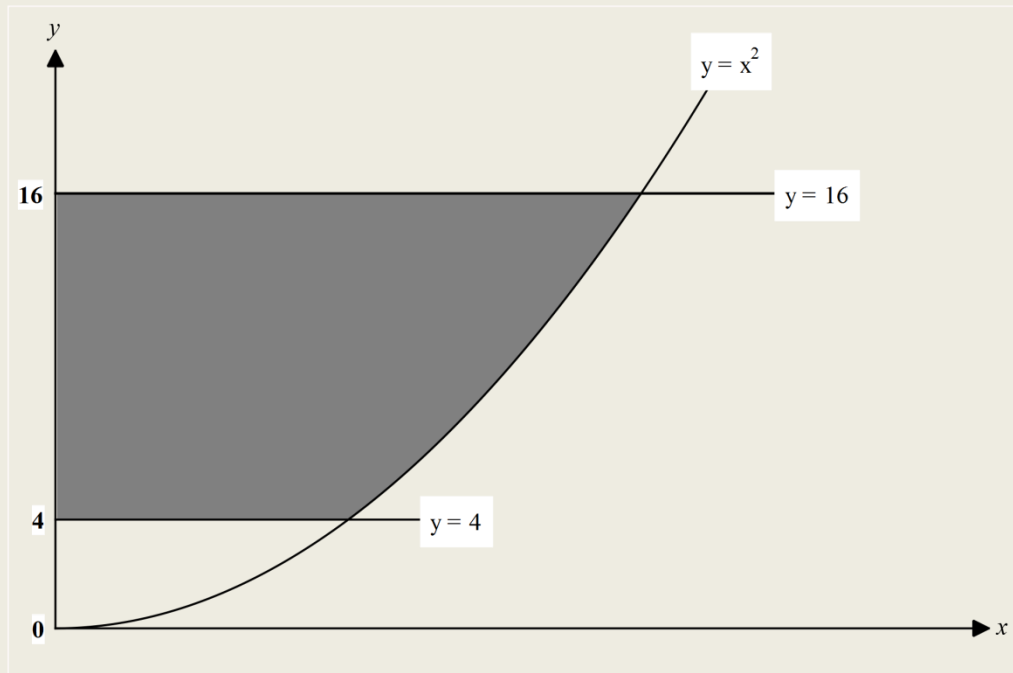
$$= 42 \text{ sq. units}$$

**Area of shaded region;**

$$= 78 - 42$$

$$= 36 \text{ sq. units.}$$

4. The shaded region in the figure below is bounded by the curve  $y = x^2$  and the line  $y = 4$  and  $y = 16$ .



(a) Calculate the exact area of the shaded region.

(4mks)

$$y = x^2$$

$$x = \sqrt{y}$$

$$x = y^{\frac{1}{2}}$$

Area of shaded;

$$= \int_4^{16} (y^{\frac{1}{2}}) dy$$

$$\left[ \frac{2}{3} y^{\frac{3}{2}} \right]_4^{16}$$

$$= \left[ \frac{2}{3} (16)^{\frac{3}{2}} \right] - \left[ \frac{2}{3} (4)^{\frac{3}{2}} \right]$$

$$= \frac{2}{3} (64) - \frac{2}{3} (8)$$

$$= \frac{128}{3} - \frac{16}{3} = \frac{112}{3} \text{ sq. units.}$$

- (b) Estimate the area of the shaded region using;  
 i. Trapezium rule and a height of 1 unit.

(3mks)

At P,  $y = 4$

$$4 = x^2$$

$$x = \sqrt{4}$$

$$x = \pm 2$$

at Q,  $y = 16$

$$x^2 = 16$$

$$x = \sqrt{16}$$

$$x = \pm 4$$

Area under line y

$$= 16 - \text{Area under line } y$$

$$= 4.$$

x	0	1	2
$y_1$	16	16	16
$y_2$	4	4	4
$y_1 - y_2$	12	12	12

$$\text{Area} = \frac{1}{2} [(12 + 12) + 2(12)]$$

$$= 24 \text{ sq. units}$$

Area from  $2 \leq x \leq 4$ .

x	2	3	4
$y_1$	16	16	16
$y_2$	4	9	16
$y_1 - y_2$	12	7	0

$$\text{Area} = \frac{1}{2} [(12 + 0) + 2(7)]$$

$$= 13 \text{ sq. units}$$

Total Area of shaded region;

$$= 24 + 13$$

$$= 37 \text{ sq. units.}$$

ii. Mid – ordinate rule and height of 1 unit.

(3mks)

**Area between lines;**

$$x = 16$$

$$\text{and } y = 4$$

**from**

$$0 \leq x \leq 2.$$

<b>x</b>	<b>0.5</b>	<b>1.5</b>
<b>y<sub>1</sub></b>	<b>16</b>	<b>16</b>
<b>y<sub>2</sub></b>	<b>4</b>	<b>4</b>
<b>y<sub>1</sub> - y<sub>2</sub></b>	<b>12</b>	<b>12</b>

$$\text{Area} = 1(12 + 12)$$

$$= 24 \text{ sq. units.}$$

**Area between lines;**

$$y = 16 \text{ and } y = x^2$$

$$\text{from } 2 \leq x \leq 4.$$

<b>x</b>	<b>2.5</b>	<b>3.5</b>
<b>y<sub>1</sub></b>	<b>16</b>	<b>16</b>
<b>y<sub>2</sub></b>	<b>6.25</b>	<b>12.25</b>
<b>y<sub>1</sub> - y<sub>2</sub></b>	<b>9.75</b>	<b>3.75</b>

$$\text{Area} = 1(9.75 + 3.75)$$

$$= 13.5 \text{ sq. units}$$

**Total shaded area;**

$$= 24 + 13.5$$

$$= 37\frac{1}{2} \text{ sq. units.}$$

5. (a) complete the table below for  $y = 3 \sin 2x$  in the range  $0^\circ \leq x \leq \frac{\pi^c}{2}$  to 4 significant figures.

$x^\circ$	0	$\frac{1}{12}\pi$	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{5}{12}\pi$	$\frac{1}{2}\pi$
y	0.0000	1.5000	2.5981	3.0000	2.5981	1.5000	0.0000

(b) Estimate the area of bounded by the curve  $y = 3 \sin 2x$  for  $0^\circ \leq x \leq \frac{\pi^c}{2}$  using;

i. Trapezium rule and 7 ordinates. (3mks)

No. of strips;

$$= 7 - 1$$

$$= 6$$

$$h = \frac{\frac{1}{2}\pi^c - 0}{6}$$

$$= \frac{1}{12}\pi^c$$

$$A = \frac{1}{24}\pi^c[(0 + 0) + 2(1.5 + 2.598 + 3 + 2.598 + 1.5)]$$

$$= \frac{1}{24}\pi^c(22.392)$$

$$= 0.933\pi^c$$

$$= 2.9311 \text{ sq. units.}$$

ii. Mid – ordinate rule and 3 strips. (3mks)

$$h = \frac{\frac{1}{2}\pi^c - 0}{3}$$

$$= \frac{1}{6}\pi^c$$

$$A = \frac{1}{6}\pi^c(1.5 + 3 + 1.5)$$

$$= \frac{1}{6}\pi^c \times 6$$

$$= \pi^c$$

$$= 3.1416 \text{ sq. units.}$$

(c) Given that  $\int_0^{\frac{1}{2}\pi} (3 \sin 2x) dx = 3$ , calculate the error in (b)(i) and (ii) above. (2mks)

Actual area;

$$= 0.02297$$

$$= 0.0472$$

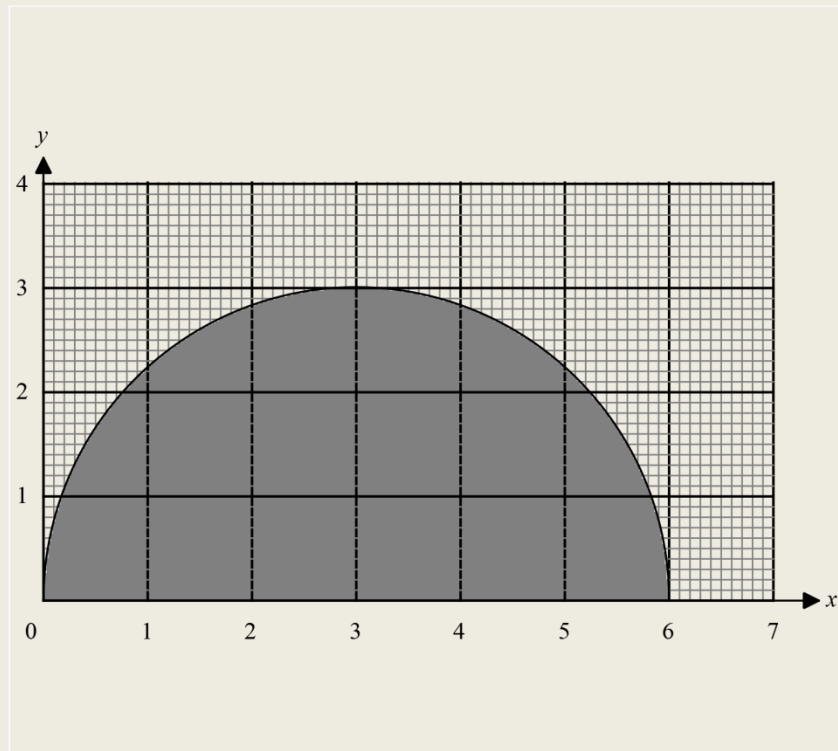
$$\int_0^{\frac{1}{2}\pi} (3 \sin 2x) dx = 3$$

or

$$\text{Error} = \frac{3 - 3.1416}{3}$$

$$\text{Error} = \frac{3 - 2.9311}{3}$$

6. The figure below shows a semi – circle centre (3,0) and radius 3 units.



(a) Estimate the area of the semi – circle using;

i. Trapezoidal rule and 6 strips.

(4mks)

<b>x</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>y</b>	<b>0</b>	<b>2.2</b>	<b>2.8</b>	<b>3</b>	<b>2.8</b>	<b>2.2</b>	<b>0</b>

$$h = \frac{6 - 0}{6}$$

$$= 1 \text{ unit}$$

$$\frac{1}{2} [(0 +) + 2(2.2 + 2.8 + 3 + 2.8 + 2.2)]$$

$$= \frac{1}{2} (26)$$

$$= 13 \text{ sq. units.}$$

ii. Mid – ordinate rule and 6 strips.

(4mks)

$$h = \frac{6 - 0}{6}$$

$$= 1 \text{ unit}$$

$$A = 1(1.6 + 2.6 + 2.95 + 2.95 + 2.6 + 1.6)$$

$$= 14.3 \text{ sq. units.}$$

<b>x</b>	<b>0.5</b>	<b>1.5</b>	<b>2.5</b>	<b>3.5</b>	<b>4.5</b>	<b>5.5</b>
<b>y</b>	<b>1.6</b>	<b>2.6</b>	<b>2.95</b>	<b>2.95</b>	<b>2.6</b>	<b>1.6</b>

- (b) Find, in terms of  $\pi$ , the error in the area of the semi – circle when mid – ordinate rule is used as in (a) (ii) above. (2mks)

**Exact Area;**

$$A = \frac{1}{2} \pi r^2$$

$$= \frac{\pi(3)^2}{2}$$

$$= \frac{9}{2} \pi$$

**Error absolute;**

$$= \left| \frac{9}{2} \pi - 14.3 \right|.$$

7. A region **R** is bounded by the curve  $y = x^3$ , the  $x -$  axis and the ordinates  $x = -3$  and  $x = 3$ . Determine;  
 (a) The exact area of the region **R**. (4mks)

**Points where the curve cuts  $x -$  axis;**

$$y = 0$$

$$x = 0$$

**so we integrate from;**

$$-3 \text{ to } 0$$

**then from 0 to 3**

$$A = 2 \times \int_{-3}^0 (x^3) dx$$

$$= 2 \times \left[ \frac{1}{4} y^4 \right]_{-3}^0$$

$$= 2 \left\{ \left[ \frac{1}{4} (0)^4 \right] - \left[ \frac{1}{4} (-3)^4 \right] \right\}$$

$$= 2 \times 20 \frac{1}{4}$$

$$= 40 \frac{1}{2} \text{ sq. units.}$$

- (b) Estimate the area of the region **R** using the mid – ordinate rule and 7 ordinates. (4mks)

$$h = \frac{3 - (-3)}{6}$$

$$= 1 \text{ unit.}$$

<b>x</b>	<b>-2.5</b>	<b>-1.5</b>	<b>-0.5</b>	<b>0.5</b>	<b>1.5</b>	<b>2.5</b>
<b>y</b>	<b>-15.625</b>	<b>-3.375</b>	<b>-0.125</b>	<b>0.125</b>	<b>3.375</b>	<b>15.625</b>

$$A = 1(15.626 + 3.375 + 0.125 + 0.125 + 3.375 + 15.625)$$

$$= 38 \frac{1}{4} \text{ sq. units.}$$

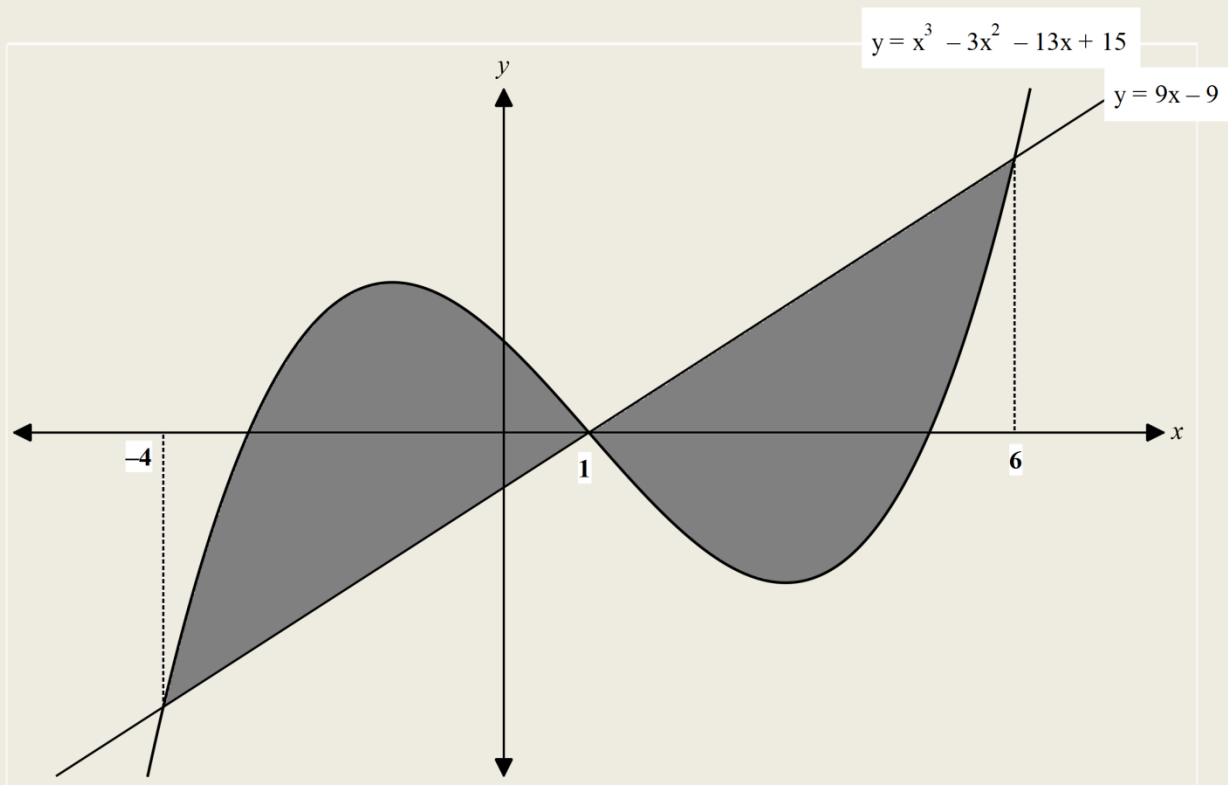
(c) Calculate the percentage error in the area of **R** in (b) above.

(2mks)

$$\begin{aligned} & \text{\% error;} \\ & = \left( \frac{40\frac{1}{2} - 38\frac{1}{4}}{40\frac{1}{2}} \right) 100 \end{aligned}$$

$$= 5\frac{5}{9}\%$$

8. In the figure below, the shaded region is bounded by the curve  $y = x^3 - 3x^2 - 13x + 15$  and the straight line  $y = 9x - 9$ .



(a) Calculate the exact area of the shaded region.

(4mks)

$$= 2 \left\{ \int_1^6 (9x - 9) dx - \int_1^6 (x^3 - 3x^2 - 13x + 15) dx \right\}$$

$$= 2 \left\{ \left[ \frac{9}{2}x^2 - 9x \right]_1^6 + \left[ \frac{x^4}{4} - x^3 - \frac{13}{2}x^2 + 15x \right]_1^6 \right\}$$

For;

$$\left[ \frac{9}{2}x^2 - 9x \right]_1^6$$

$$= \left[ \frac{9}{2}(6)^2 - 9(6) \right]$$

$$- \left[ \frac{9}{2}(1)^2 - 9(1) \right]$$

$$= \frac{225}{2}$$

For;

$$\left[ \frac{x^4}{4} - x^3 - \frac{13}{2}x^2 + 15x \right]_1^6$$

$$= \left[ \frac{(4)^4}{4} - (4)^3 - \frac{13}{2}(4)^2 + 15(4) \right]$$

$$- \left[ \frac{(1)^4}{4} - (1)^3 - \frac{13}{2}(1)^2 + 15(1) \right]$$

$$= -\frac{175}{4}$$

$$2 \left\{ \frac{225}{2} - \left( -\frac{175}{4} \right) \right\}$$

$$= 312.5 \text{ sq. units.}$$

(b) Estimate the area of the shaded region using trapezium rule with 10 strips. (4mks)

Consider from intersection point towards R. H. S.

For the curve  $y = x^3 - 3x^2 - 13x + 15$ .

x	1	2	3	4	5	6
y	0	-15	-24	-21	0	0

Consider from intersection point towards R. H. S.

For the curve  $y = 9x - 9$ .

x	1	2	3	4	5
y	0	9	18	27	36

Taking absolute value for  $y_T$

$$y_1 = (0 + 0)$$

$$= 0$$

$$y_2 = (15 + 9)$$

$$= 24$$

$$y_3 = (24 + 18)$$

$$= 42$$

$$y_4 = (21 + 27)$$

$$= 48$$

$$y_5 = (0 + 36)$$

$$= 36$$

$$y_6 = 0$$

$$A = 2 \left[ \frac{1}{2} \{ (0 + 0) + 2(24 + 42 + 48 + 36) \} \right]$$

$$= 2[0 + 150]$$

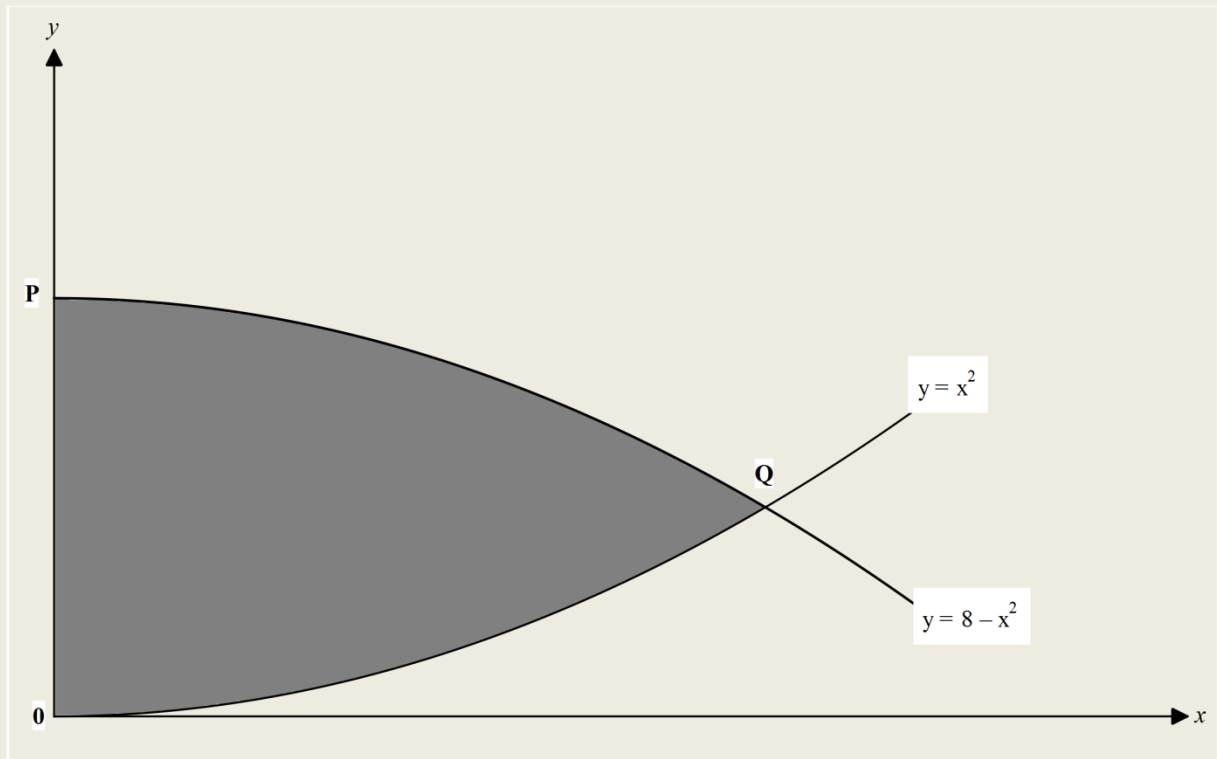
$$= 300 \text{ sq. units.}$$

(c) Hence determine the percentage error in the area in (b) above.

(2mks)

$$\begin{aligned} & \text{\% error;} \\ & = \left( \frac{312.5 - 300}{312.5} \right) 100 \\ & = 4\%. \end{aligned}$$

9. In the figure below, the curve  $y = x^2$  and  $y = 8 - x^2$  intersect at Q.



(a) Determine the coordinates of Q.

(1mk)

**At point of intersection, Q;**

$$x^2 = 8 - x^2$$

$$2x^2 = 8$$

$$x^2 = 4$$

$$x = \sqrt{4}$$

$$x = \pm 2$$

**Point:**

$$y = 8 - (2)^2$$

$$Q(2, 4)$$

(b) By integration, calculate the area of the shaded region.

(3mks)

**area under curve;**

$$y = 8 - x^2$$

$$A = \int_0^2 (8 - x^2) dx$$

$$= \left[ 8x - \frac{1}{3}x^3 \right]_0^2$$

$$= \left[ 8(2) - \frac{1}{3}(2)^3 \right] - \left[ 8(0) - \frac{1}{3}(0)^3 \right]$$

$$= \frac{40}{3} - 0$$

$$= \frac{40}{3}$$

**Area under the curve;**

$$y = x^2$$

$$= \int_0^2 (x^2) dx$$

$$= \left[ \frac{1}{3}x^3 \right]_0^2$$

$$= \left[ \frac{1}{3}(2)^3 \right] - \left[ \frac{1}{3}(0)^3 \right]$$

$$= \frac{8}{3}$$

**Area of the shaded region;**

$$= \frac{40}{3} - \frac{8}{3}$$

$$= 10 \frac{2}{3} \text{ sq. units.}$$

(c) Estimate the area of the shaded region using;

(i) Trapezium rule with 4 strips.

(3mks)

$$h = \frac{2 - 0}{4}$$

$$= 0.5 \text{ units.}$$

<b>x</b>	<b>0</b>	<b>0.5</b>	<b>1</b>	<b>1.5</b>	<b>2</b>
<b><math>y_1 = 8 - x^2</math></b>	<b>8</b>	<b>7.75</b>	<b>7</b>	<b>5.75</b>	<b>4</b>
<b><math>y_2 = x^2</math></b>	<b>0</b>	<b>0.25</b>	<b>1</b>	<b>2.25</b>	<b>4</b>

**Area under the curve;**

$$y = 8 - x^2$$

$$= \frac{0.5}{2} [(8 + 4) + 2(7.75 + 7 + 5.75)]$$

$$= 13.25 \text{ sq. units}$$

**Area under the curve;**

$$y = x^2$$

$$= \frac{0.5}{2} [(0 + 4) + 2(0.25 + 1 + 2.25)]$$

$$= 2.75 \text{ sq. units}$$

**Area of the shaded region;**

$$= 13.25 - 2.75$$

$$= 10 \frac{1}{2} \text{ sq. units.}$$

(ii) Mid – ordinate rule with 4 strips.

(3mks)

$$h = \frac{2 - 0}{4}$$

$$= 0.5 \text{ units}$$

x	0.25	0.75	1.25	1.75
$y_1 = 8 - x^2$	7.9375	7.4375	6.4375	4.9375
$y_2 = x^2$	0.0625	0.5625	1.5625	3.0625

Area under curve;

$$y = 8 - x^2$$

$$= 0.5(7.9375 + 7.4375 + 6.4375 + 4.9375)$$

$$= 0.5 \times 26.75$$

$$= 13.375 \text{ sq. units}$$

Area under curve;

$$y = x^2$$

$$= 0.5(0.0625 + 0.5625 + 1.5625 + 3.0625)$$

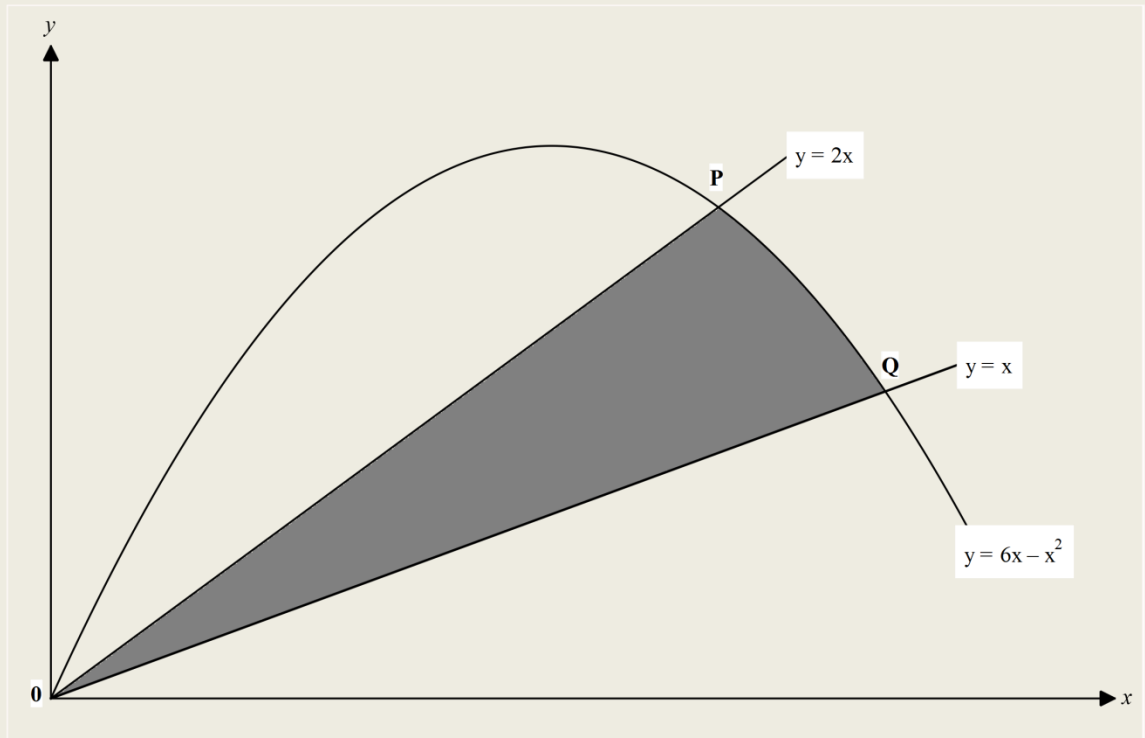
$$= 2.625 \text{ sq. units}$$

Area of the shaded region;

$$= 13.375 - 2.625$$

$$= 10\frac{3}{4} \text{ sq. units.}$$

10. In the figure below, the shaded region is bounded by the lines  $y = 2x$ ,  $y = x$  and the curve  $y = 6x - x^2$ . The two straight lines intersect the curve at points P and Q respectively.



(a) Determine the coordinates of points P and Q.

(2mks)

**Points of intersection;**

$$6x - x^2 = 2x$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$x = 0$$

$$x = 4$$

**at  $x = 4$ ;**

$$y = 2(4)$$

$$= 8$$

$$\mathbf{P(4, 8)}$$

**Also;**

$$6x - x^2 = x$$

$$x^2 - 5x = 0$$

$$x(x - 5) = 0$$

$$x = 0$$

$$x = 5$$

**at  $x = 5$ ;**

$$y = 5$$

$$\mathbf{Q(5, 5)}$$

(b) Calculate the exact area of the shaded region.

(4mks)

**Area under the curve;**

$$\begin{aligned}y &= 6x - x^2 \\&= \int_0^5 (6x - x^2) dx \\&= \left[ 3x^2 - \frac{1}{3}x^3 \right]_0^5 \\&= \left[ 3(5)^2 - \frac{1}{3}(5)^3 \right] - \left[ 3(0)^2 - \frac{1}{3}(0)^3 \right] \\&= 33\frac{1}{3} - 0 \\&= 33\frac{1}{3} \text{ sq. units}\end{aligned}$$

**Area under line;**

$$\begin{aligned}y &= x \\&= \int_0^5 (x) dx \\&= \left[ \frac{1}{2}x^2 \right]_0^5 \\&= \left[ \frac{1}{2}(5)^2 \right] - \left[ \frac{1}{2}(0)^2 \right] \\&= 12.5 \text{ sq. units}\end{aligned}$$

**Area under line;**

$$\begin{aligned}y &= 2x \\&= \int_0^4 (2x) dx \\&= \left[ x^2 \right]_0^4 \\&= [(4)^2] - [(0)^2] \\&= 16 \text{ sq. units}\end{aligned}$$

**Area under curve;**

$$\begin{aligned}y &= 6x - x^2 \\&= \int_0^4 (6x - x^2) dx \\&= \left[ 3x^2 - \frac{1}{3}x^3 \right]_0^4 \\&= \left[ 3(4)^2 - \frac{1}{3}(4)^3 \right] - \left[ 3(0)^2 - \frac{1}{3}(0)^3 \right] \\&= 26\frac{2}{3} - 0 \\&= 26\frac{2}{3} \text{ sq. units}\end{aligned}$$

**Area bounded by  $y = 6x - x^2$  and  $y = 2x$ ;**

$$\begin{aligned}&= 26\frac{2}{3} - 16 \\&= 10\frac{2}{3} \text{ sq. units}\end{aligned}$$

**Area bounded by  $y = 6x - x^2$  and  $y = x$ ;**

$$\begin{aligned}&= 33\frac{1}{3} - 12\frac{1}{2} \\&= 20\frac{5}{6}\end{aligned}$$

**Area of the shaded region;**

$$\begin{aligned}&= 20\frac{5}{6} - 10\frac{2}{3} \\&= 10\frac{1}{6}\end{aligned}$$

(c) Taking the height of each trapezium as 1 unit, estimate the area of the shaded region. (4mks)

<b>x</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b><math>y_1 = 6x - x^2</math></b>	<b>0</b>	<b>5</b>	<b>8</b>	<b>9</b>	<b>8</b>
<b><math>y_2 = 2x</math></b>	<b>0</b>	<b>2</b>	<b>4</b>	<b>6</b>	<b>8</b>

**Area of curve;**

$$y = 6x - x^2$$

$$= \frac{1}{2} [(0 + 8) + 2(5 + 8 + 9)]$$

$$= 26 \text{ sq. units}$$

**Area of the curve;**

$$y = 2x$$

$$\frac{1}{2} [(0 + 8) + 2(2 + 4 + 6)]$$

$$= 16 \text{ sq. units}$$

**Area bounded by  $y = 6x - x^2$  and  $y$**

$$= 2x;$$

$$= 26 - 16$$

$$= 10 \text{ sq. units}$$

**For  $y = 6x - x^2$  and  $y = x$**

<b>x</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
<b><math>y_1 = 6x - x^2</math></b>	<b>0</b>	<b>5</b>	<b>8</b>	<b>9</b>	<b>8</b>	<b>5</b>
<b><math>y_2 = 2x</math></b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>

**Area of curve;**

$$y = 6x - x^2$$

$$= \frac{1}{2} [(0 + 5) + 2(5 + 8 + 9 + 8)]$$

$$= 32.5 \text{ sq. units}$$

**Area of the curve;**

$$y = x$$

$$\frac{1}{2} [(0 + 5) + 2(1 + 2 + 3 + 4)]$$

$$= 12.5 \text{ sq. units}$$

**Area bounded by  $y = 6x - x^2$  and  $y$**

$$= x;$$

$$= 32.5 - 12.5$$

$$= 20 \text{ sq. units}$$

**Area of the shaded region;**

$$= 20 - 10$$

$$= 10 \text{ sq. units.}$$

# Differentiation and Integration.

1. The curve  $y = x^3 - 2x^2 + kx + 12$  where  $k$  is a constant passes through a point  $P(2, -10)$ .  
(a) Find the value of  $k$ . (2mks)

$y = x^3 - 2x^2 + kx + 12$	$-10 + 12 + 2k$
$P(2, -10)$	$2k = -22$
$-10 = (2)^3 - 2(2)^2 + k(2) + 12$	$k = -11.$

- (b) Find the equation of a normal to the curve at  $P$ . (4mks)

$\frac{dy}{dx} = 3x^2 - 4x - 11$	$= \frac{1}{7}$
at $x = 2$ ;	Equation;
Grad. = $3(2)^2 - 4(2) - 11$	$\frac{y + 10}{x - 2} = \frac{1}{7}$
$= -7$	$7y + 70 = x - 2$
Grad. of tangent = $-7$	$7y - x + 72 = 0.$
Grad of normal;	

(c) Investigate the turning points of the curve.

(4mks)

At turning point  $\frac{dy}{dx} = 0$

$$3x^2 - 4x - 11 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-11)}}{2(3)}$$

$$x = \frac{4 \pm 12.1655}{6}$$

$$x = 2.694$$

or

$$x = -1.362$$

When  $x = 2.694$

$$y = (2.694)^3 - 2(2.694)^2 - 11(2.694) + 12$$

$$y = -12.597$$

Turning point;

(2.694, -12.597)

Nature;

x	2	2.694	3
$\frac{dy}{dx}$	-4	0	4
Nature			

Point (2.694, -12.597);

Is a minimum turning point

When  $x = -1.362$

$$y = (-1.362)^3 - 2(-1.362)^2 - 11(-1.362) + 12$$

$$y = 20.475$$

Turning point;

(-1.362, 20.475)

Nature;

$$\frac{d^2y}{d^2x} = 6(-1.361) - 4$$

$$= -12.166$$

point;

(-1.362, 20.475);

Is a maximum turning point

2. A curve is represented by the function  $y = x^3 - 4x^2 + 5x - 2$ .

(a) Find  $\frac{dy}{dx}$ .

(1mk)

$$y = x^3 - 4x^2 + 5x - 2$$

$$\frac{dy}{dx} = 3x^2 - 8x + 5.$$

(b) (i) Determine the stationary points of the curve.

(4mks)

At stationary point  $\frac{dy}{dx} = 0$

$$3x^2 - 8x + 5 = 0$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(5)}}{2(3)}$$

$$= \frac{8 \pm 2}{6}$$

$$x = \frac{5}{3}$$

or

$$x = 1$$

When  $x = 1$ ;

$$y = (1)^3 - 4(1)^2 + 5(1) - 2 = 0$$

Turning point;

P(1, 0)

When  $x = \frac{5}{3}$ ;

$$y = \left(\frac{5}{3}\right)^3 - 4\left(\frac{5}{3}\right)^2 + 5\left(\frac{5}{3}\right) - 2 = -0.1481$$

Turning point;

Q  $\left(\frac{5}{3}, -0.1481\right)$ .

(ii) For each stationary points, determine whether it is minimum, a maximum or a point of inflection.

(2mks)

Using Second derivative;

$$\frac{d^2y}{d^2x} = 6x - 8$$

At  $x = 1$ ;

$$\frac{d^2y}{d^2x} = -2$$

P(1, 0);

Is a maximum turning point

When  $x = \frac{5}{3}$

$$\frac{d^2y}{d^2x} = 2$$

Q  $\left(\frac{5}{3}, -0.1481\right)$ ;

Is a minimum turning point.

(c) In the space provided, sketch the curve of the function  $y = x^3 - 4x^2 + 5x - 2$ . (3mks)

**y – intercept;**

$$x = 0$$

$$y = (0)^3 - 4(0)^2 + 5(0) - 2$$

$$= -2$$

**Coordinates (0, -2)**

**x – intercept;**

$$y = 0$$

$$x^3 - 4x^2 + 5x - 2 = 0$$

**x = 1 satisfy the equation;**

$$(x - 1) \begin{array}{r} x^2 - 3x + 2 \\ \hline x^3 - 4x^2 + 5x - 2 \\ x^3 - x^2 \phantom{+ 5x - 2} \\ \hline -3x^2 + 5x - 2 \\ -3x^2 + 3x \phantom{- 2} \\ \hline 2x - 2 \\ - 2x - 2 \\ \hline 0 \end{array}$$

**Reduced equation;**

$$= (x - 1)(x^2 - 3x + 2) = 0$$

**Consider;**

$$x^2 - 3x + 2 = 0$$

$$p = 2x^2$$

$$s = -3x$$

$$f = -2x \text{ and } -x$$

$$x^2 - 2x - x + 2 = 0$$

$$x(x - 2) - 1(x - 2) = 0$$

$$(x - 1)(x - 2) = 0$$

**Generally;**

$$(x - 1)(x - 1)(x - 2) = 0$$

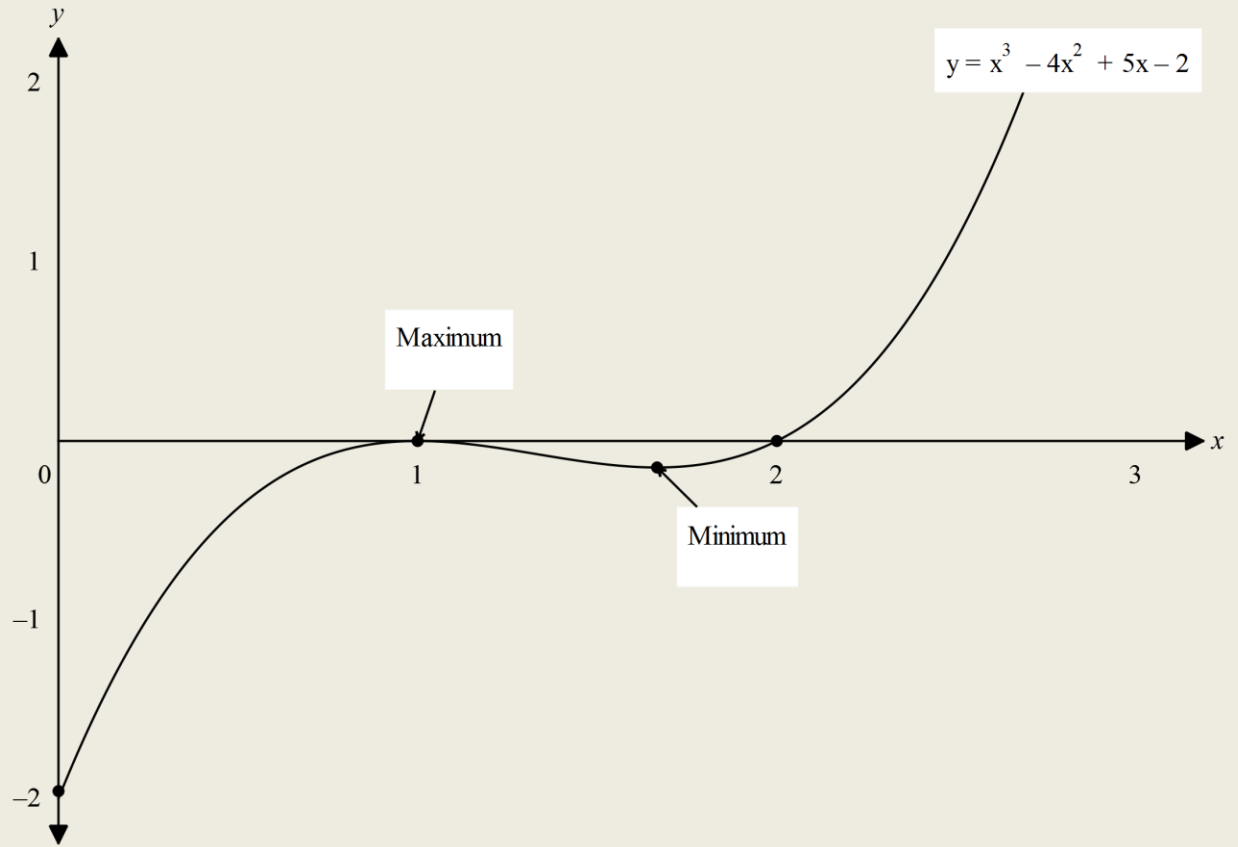
$$x = 1$$

**or**

$$x = 2$$

**Coordinates;**

**(1, 0) and (2, 0)**



3. The equation of a curve is  $y = -3x^2 - x^3$ .

(a) Find;

(i) The x – intercepts of the curve.

(2mks)

$$y = -3x^2 - x^3$$

At x – intercept;

$$y = 0$$

$$-3x^2 - x^3 = 0$$

$$-x^2(3 + x) = 0$$

$$x = 0$$

or

$$3 + x = 0$$

$$x = -3$$

Coordinates;

(0, 0) and (-3, 0).

(ii) The y – intercept of the curve.

(1mk)

At y – intercept;

$$x = 0$$

$$y = -3(0)^2 - (0)^3$$

$$= 0$$

Coordinates;

(0, 0).

(b) (i) Determine the stationary points of the curve.

(3mks)

At stationary point;

$$\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -6x - 3x^2$$

$$-6x - 3x^2 = 0$$

$$-3x(2 + x) = 0$$

$$x = 0$$

or

$$x = -2$$

When  $x = 0$

$$y = -3(0)^2 - (0)^3$$

$$= 0$$

Turning points;

$$= (0, 0)$$

At  $x = -2$

$$y = -3(-2)^2 - (-2)^3$$

$$= -4$$

Turning points;

(-2, -4).

(ii) For each point in (b) (i) above, determine whether it is a maximum or a minimum. (2mks)

Using second derivative;

$$\frac{d^2y}{d^2x} = -6 - 6x$$

$$\text{At } x = 0$$

$$\frac{d^2y}{d^2x} = -6$$

Point (0, 0);

Is a maximum turning point

$$\text{At } x = -2;$$

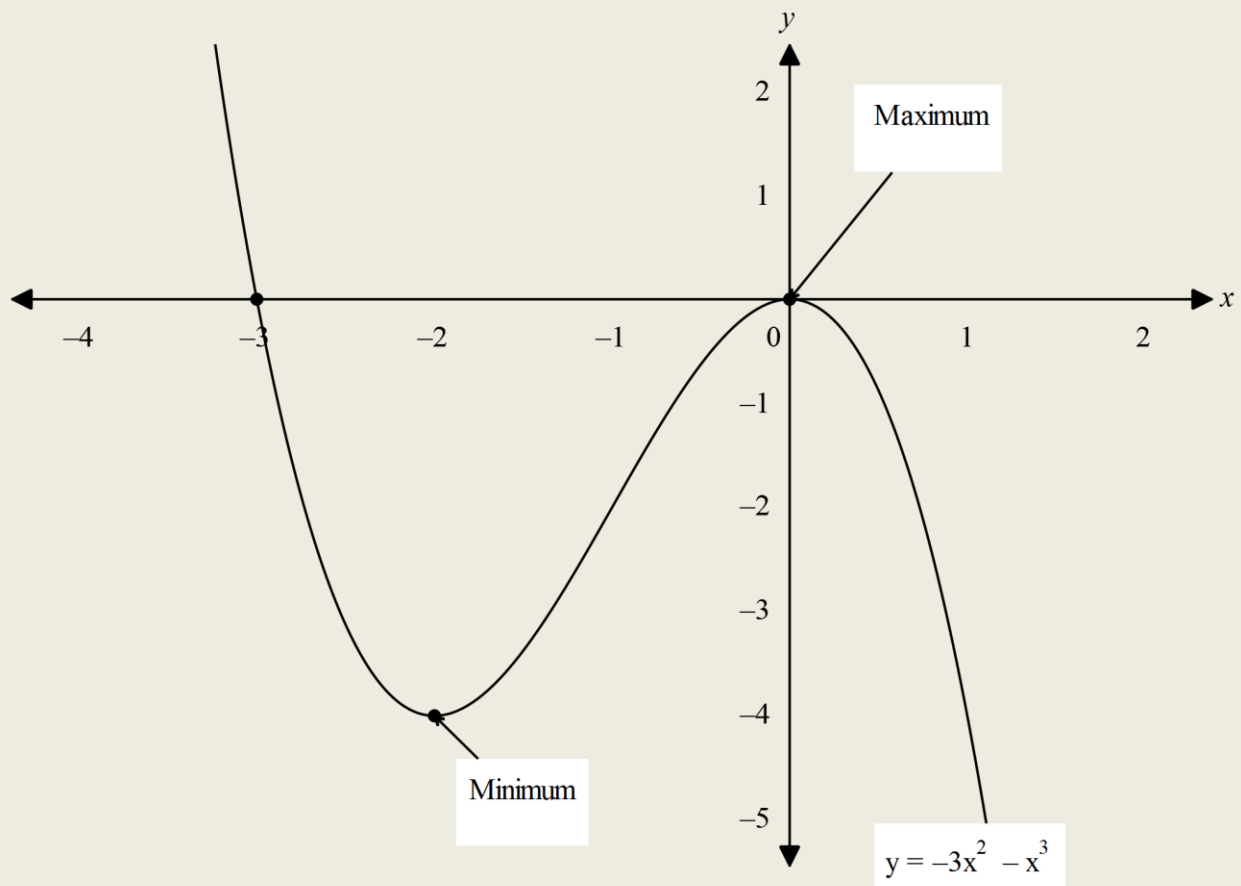
$$\frac{d^2y}{d^2x} = 6$$

Point (-2, -4);

Is a minimum turning point.

(c) Sketch the curve.

(2mks)



4. The equation of the curve is given by  $y = x^3 - 6x^2 + 9x$ .

Find;

(a) The coordinates of the x – intercepts.

(3mks)

$$y = x^3 - 6x^2 + 9x$$

At x – intercept;

$$y = 0$$

$$x(x^2 - 6x + 9) = 0$$

$$x = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{6 \pm 0}{2}$$

$$x = 3$$

x – intercept;

(0, 0) and (3, 0).

(b) The coordinates of the stationary points of the curve.

(4mks)

At stationary point;

$$\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 3x^2 - 12x + 9$$

$$3x^2 - 12x + 9 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2}$$

$$x = \frac{4 \pm 2}{2}$$

$$x = 3$$

or

$$x = 1$$

At x = 3

$$y = (3)^3 - 6(3)^2 + 9(3) \\ = 0$$

Turning points;

(1, 0)

At x = 1

$$y = (1)^3 - 6(1)^2 + 9(1) \\ = 4$$

Turning points;

(1, 4).

(c) Clearly sketch the curve.

(3mks)

**At y – intercept;**

$$x = 0$$

$$y = (0)^3 - 6(0)^2 + 9(0)$$

$$y = 0$$

**Coordinates;**

$$(0, 0)$$

**Nature;**

**Using second derivative;**

$$\frac{d^2y}{d^2x} = 6x - 12$$

$$\text{At } x = 3$$

$$\frac{d^2y}{d^2x} = 6$$

**Point (3, 0)**

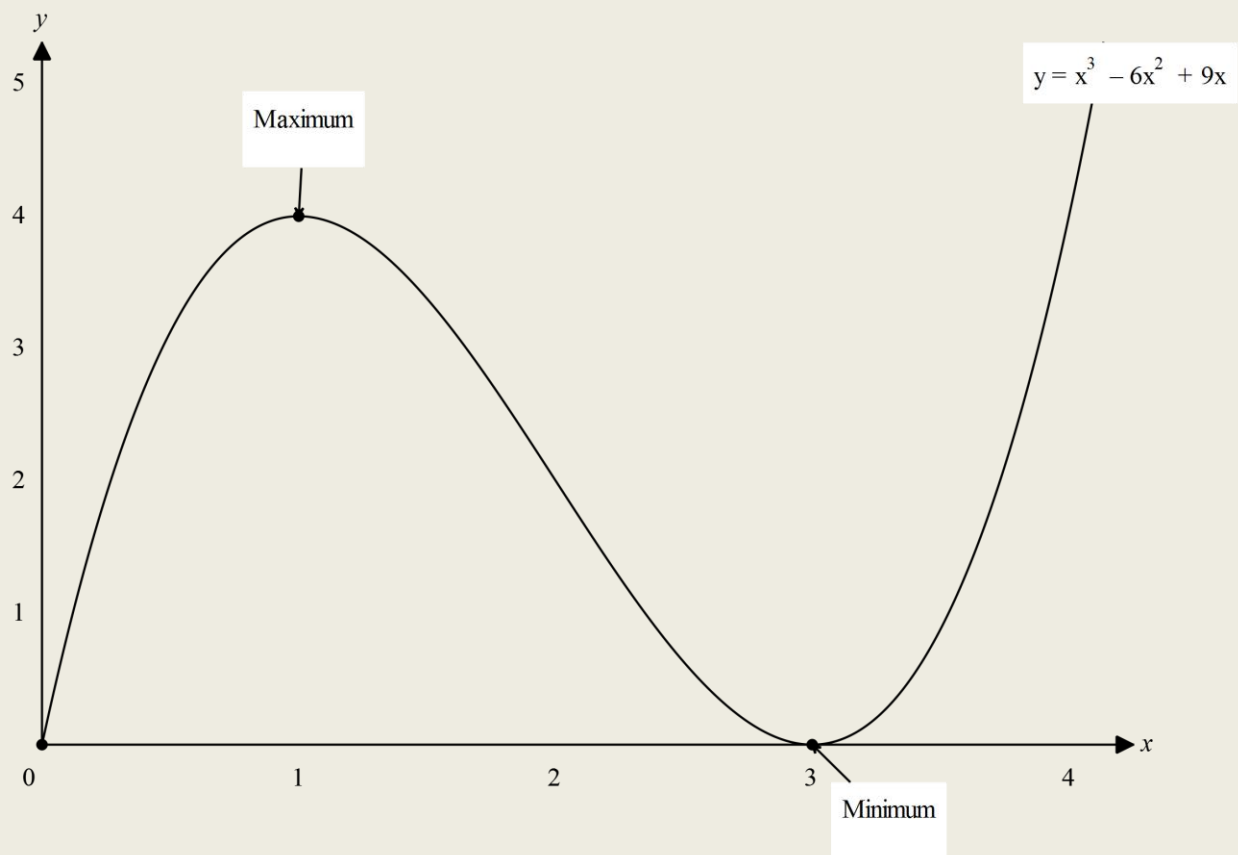
**Is a minimum turning point**

$$\text{At } x = 1$$

$$\frac{d^2y}{d^2x} = -6$$

**Point (1, 4);**

**Is a maximum turning point.**



5. The gradient function of a curve is given by  $\frac{dy}{dx} = 3x^2 + 12x - 15$ . The curve passes through the point P(-1,4).

(a) Determine the equation of the curve. (4mks)

$$\frac{dy}{dx} = 3x^2 + 12x - 15$$

$$y = \int (3x^2 + 12x - 15) dx$$

$$= x^3 + 6x^2 - 15x + c$$

At P(-1, 4)

$$4 = (-1)^3 + 6(-1)^2 - 15(-1) + c$$

$$4 = 20 + c$$

$$c = -16$$

$$y = x^3 + 6x^2 - 15x - 16.$$

(b) Find the stationary points of the curve, and for each point, determine its nature. (3mks)

At stationary points;

$$\frac{dy}{dx} = 0$$

$$3x^2 + 12x - 15 = 0$$

$$x^2 + 4x - 5 = 0$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{-4 \pm 6}{2}$$

$$x = 1$$

or

$$x = -5$$

When  $x = 1$

$$y = (1)^3 + 6(1)^2 - 15(1) - 16$$

$$= -24$$

Turning point;

$$= (1, -24)$$

When  $x = -5$

$$y = (-5)^3 + 6(-5)^2 - 15(-5) - 16$$

$$= 84$$

Turning point;

$$= (-5, 84)$$

Nature;

$$\frac{d^2y}{d^2x} = 6x + 12$$

$$\text{at } x = 1$$

$$\frac{d^2y}{d^2x} = 18$$

Point (1, -24);

Is a minimum turning point

$$\text{At } x = -5$$

$$\frac{d^2y}{d^2x} = -18$$

Point (-5, 84);

Is a maximum turning point.

(c) Find the equation of the tangent to the curve at P.

(3mks)

$$\frac{dy}{dx} = 3x^2 + 12x - 15$$

$$\text{at } x = -1$$

**Gradient;**

$$= 3(-1)^2 + 12(-1) - 15$$

$$= -24$$

**Equation;**

$$\frac{y - 4}{x + 1} = -24$$

$$y - 24x - 24 + 4$$

$$y = -24x - 20.$$

6. The gradient of a curve at a point (x, y) is  $4x - 3$ . The curve has a minimum value of  $-\frac{1}{8}$ .

(a) Find;

(i) The value of x at the minimum point.

(1mk)

$$\frac{dy}{dx} = 4x - 3$$

**At minimum point;**

$$\frac{dy}{dx} = 0$$

$$4x - 3 = 0$$

$$x = \frac{3}{4}.$$

(ii) The equation of the curve.

(4mks)

$$y = \int (4x - 3) dx$$

$$= 2x^2 - 3x + c$$

$$\text{at } \left(\frac{3}{4}, -\frac{1}{8}\right)$$

$$-\frac{1}{8} = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) + c$$

$$c - \frac{9}{8} = -\frac{1}{8}$$

$$= 1$$

$$y = 2x^2 - 3x + 1.$$

- (b) P is a point on the curve in part (a) (ii) above. If the gradient of the curve at P is  $-7$ , find the coordinates of P. (3mks)

<p style="text-align: center;"><b>At P;</b></p> $4x - 3 = -7$ $4x = -4$ $x = -1$		$y = 2(-1)^2 - 3(-1) + 1$ $y = 6$ $P(-1, 6).$
--	--	---

- (c) Find the equation of the normal to the curve at P. (2mks)

<p style="text-align: center;"><b>Gradient of tangent = <math>-7</math></b></p> <p style="text-align: center;"><b>Gradient of normal = <math>\frac{1}{7}</math></b></p> <p style="text-align: center;"><b>Equation of normal;</b></p>		$\frac{y - 6}{x + 1} = \frac{1}{7}$ $7y - 42 = x + 1$ $7y - x - 43 = 0.$
---	--	--

7. The gradient of a curve is given as  $\frac{dy}{dx} = x^2 - x + a$  where a is a constant. If one of the turning points of the curve is  $(-3, 32\frac{1}{2})$ , determine;

- (a) The value of a. (2mks)

$\frac{dy}{dx} = x^2 - x + a$ <p style="text-align: center;"><b>At the turning point;</b></p> $\frac{dy}{dx} = 0$ $x^2 - x + a = 0$ <p style="text-align: center;"><b>at <math>x = -3</math></b></p>		$(-3)^2 - (-3) + a = 0$ $9 + 3 + a = 0$ $a = -12.$
--	--	--

(b) The equation of the curve.

(3mks)

$$y = \int (x^2 - x - 12) dx$$

$$y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 12x + c$$

$$\text{At } \left(-3, 32\frac{1}{2}\right);$$

$$32\frac{1}{2} = \frac{1}{3}(-3)^3 - \frac{1}{2}(-3)^2 - 12(-3) + c$$

$$32\frac{1}{2} = 22\frac{1}{2} + c$$

$$c = 10$$

$$y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 12x + 10$$

(c) The other turning point.

(3mks)

**At turning point;**

$$\frac{dy}{dx} = 0$$

$$x^2 - x - 12 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-11)}}{2(1)}$$

$$x = \frac{1 \pm 7}{2}$$

$$x = -3$$

**or**

$$x = 4$$

$$y = \frac{1}{3}(4)^3 - \frac{1}{2}(4)^2 - 12(4) + 10$$

$$y = -24\frac{2}{3}$$

**2<sup>nd</sup> Turning point;**

$$\left(4, -24\frac{2}{3}\right).$$

(d) The nature of the turning points of the curve.

(2mks)

**Nature;**

**At  $x = 4$**

**Using second derivative;**

$$\frac{d^2y}{d^2x} = 7$$

$$\frac{d^2y}{d^2x} = 2x - 1$$

**Point  $(4, -24\frac{2}{3})$ ;**

**at  $x = -3$**

**Is a minimum turning point.**

$$\frac{d^2y}{d^2x} = -7$$

**Point  $(-3, 32\frac{1}{2})$ ;**

**Is a maximum turning point**

8. A curve is represented by the function  $y = \frac{1}{3}x^3 + x^2 - 3x + 2$ .

(a) Find the  $y$  – intercepts of the curve.

(1mk)

$$y = \frac{1}{3}x^3 + x^2 - 3x + 2$$

**= 2**

**At  $y$  – intercept;**

**Coordinates;**

**$x = 0$**

**$(0, 2)$ .**

$$y = \frac{1}{3}(0)^3 + (0)^2 - 3(0) + 2$$

(b) Determine;

(i)  $\frac{dy}{dx}$ .

(1mk)

$$y = \frac{1}{3}x^3 + x^2 - 3x + 2$$

$$\frac{dy}{dx} = x^2 + 2x - 3.$$

(ii) The turning points of the curve.

(4mks)

**At turning point;**

$$\frac{dy}{dx} = 0$$

$$x^2 + 2x - 3 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{-2 \pm 4}{2}$$

$$x = 1$$

or

$$x = -3$$

$$\text{At } x = 1$$

$$y = \frac{1}{3}(1)^3 + (1)^2 - 3(1) + 2$$

$$= \frac{1}{3}$$

**Turning point;**

$$\left(1, \frac{1}{3}\right)$$

$$\text{At } x = -3$$

$$y = \frac{1}{3}(-3)^3 + (-3)^2 - 3(-3) + 2$$

$$= 11$$

**Turning point;**

$$(-3, 11).$$

(c) For each turning point, determine whether it is a maximum or a minimum. (3mks)

Using second derivative;

$$\frac{d^2y}{d^2x} = 2x + 2$$

$$\text{At } x = 1$$

$$\frac{d^2y}{d^2x} = 4$$

$$\text{Point } \left(1, \frac{1}{3}\right);$$

Is a minimum turning point

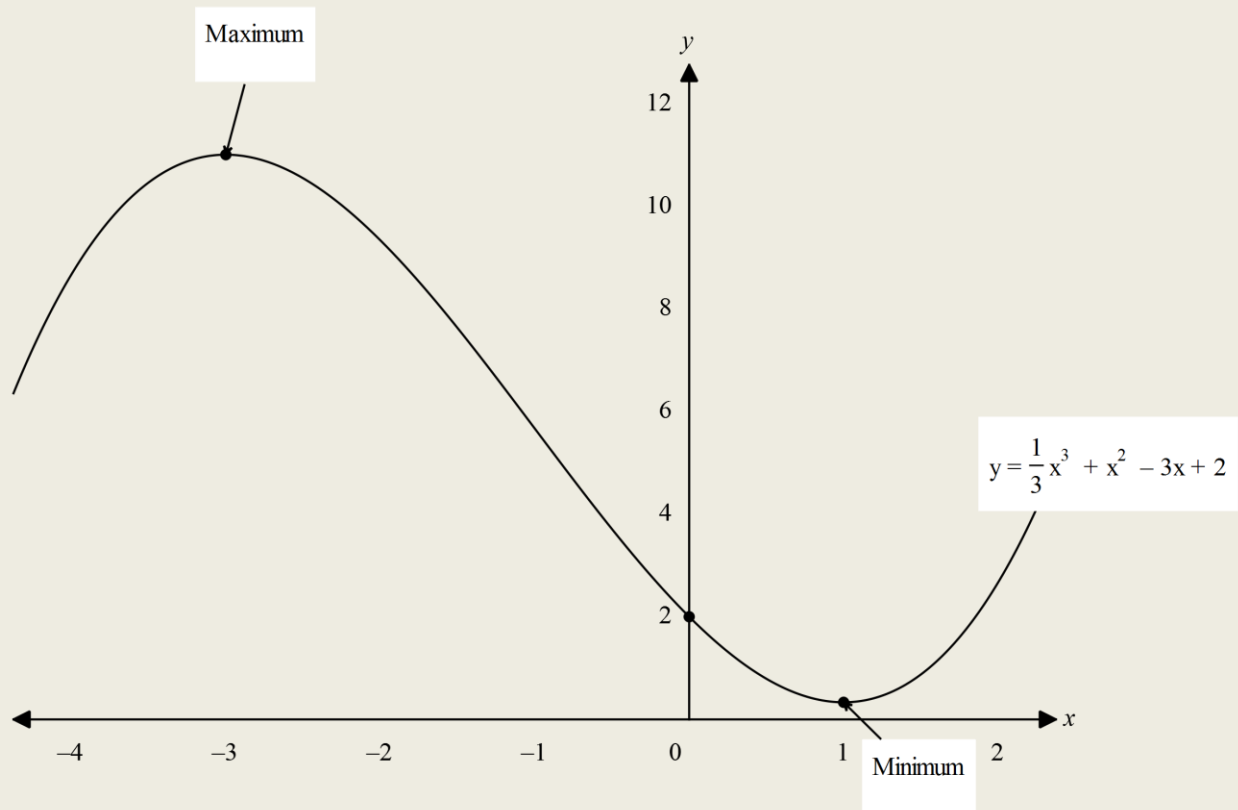
$$\text{At } x = -3$$

$$\frac{d^2y}{d^2x} = -4$$

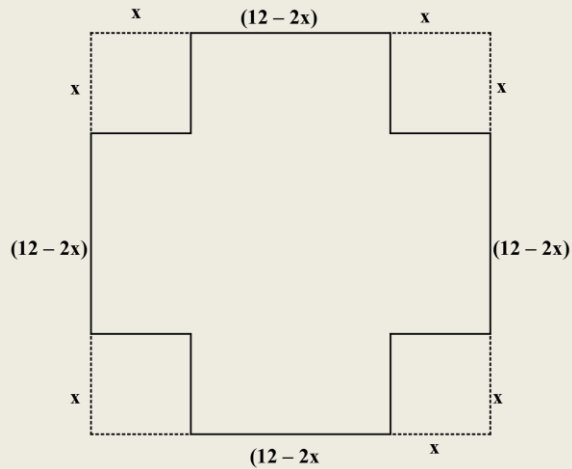
$$\text{Point } (-3, 11);$$

Is a maximum turning point.

(d) Sketch the curve. (1mk)



9. A square sheet of a paper measuring 12 cm by 12 cm has a small squares removed from each corner and then folded to form an open box. Taking the side of each squares removed as  $x$  cm;
- (a) Find an expression of the volume  $V$  of the box in terms of  $x$ . (2mks)



$$\begin{aligned}
 V &= l \times w \times h \\
 &= (12 - 2x)(12 - 2x)(x) \\
 V &= x(144 - 48x + 4x^2) \\
 V &= 144x - 48x^2 + 4x^3 \\
 V &= 4x^3 - 48x^2 + 144x.
 \end{aligned}$$

(b) Find;

- (i) The dimensions of the box for which the volume is maximum. (4mks)

**At maximum Volume;**

$$\frac{dV}{dx} = 0$$

$$\frac{dV}{dx} = 12x^2 - 96x + 144$$

$$12x^2 - 96x + 144 = 0$$

$$x^2 - 8x + 12 = 0$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(12)}}{2(1)}$$

$$x = \frac{8 \pm 4}{2}$$

$$x = 6 \text{ cm}$$

**or**

$$x = 2 \text{ cm}$$

$x \neq 6 \text{ cm}$  since width cannot be 0 cm

$$\therefore x = 2 \text{ cm}$$

**Length of box;**

$$= 12 - 2(2)$$

$$= 8 \text{ cm}$$

**Height =  $x$**

$$= 2 \text{ cm}$$

**Dimensions measures;**

**8 cm by 8 cm by 2 cm.**

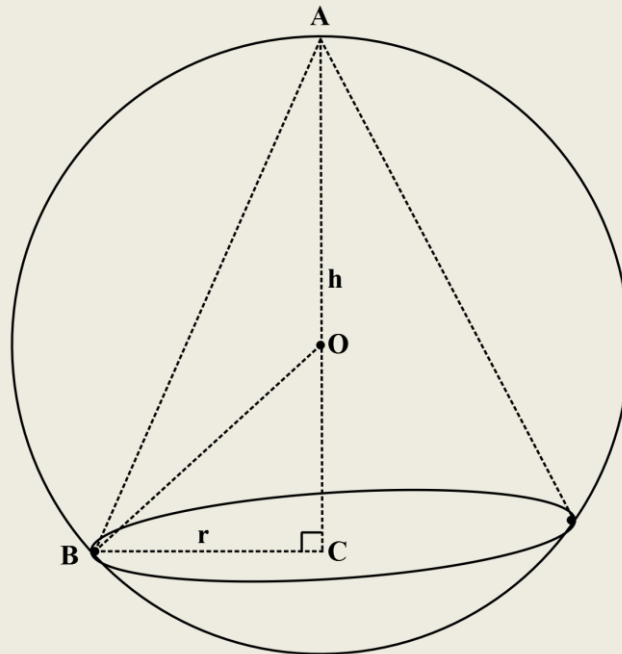
(ii) The maximum volume of the box. (2mks)

$$\begin{aligned}V_{\max} &= 8 \times 8 \times 2 \\ &= 128 \text{ cm}^3.\end{aligned}$$

(c) Calculate the surface area of the open box. (3mks)

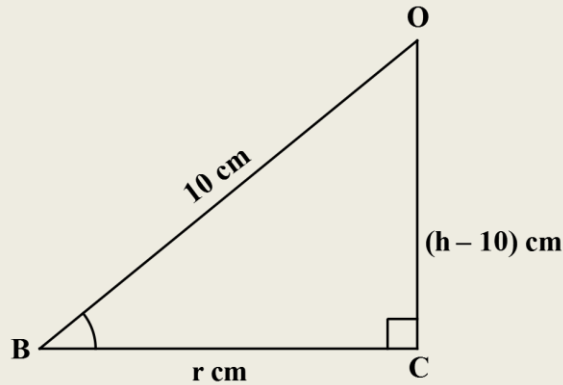
$A_1 + (8 \times 2)2$	$= 64 \text{ cm}^2$
$= 32 \text{ cm}^2$	<b>Total S. A;</b>
$A_2 = (8 \times 2)2$	$= 32 + 32 + 68$
$= 32 \text{ cm}^2$	$= 128 \text{ cm}^2.$
$A_3 = (8 \times 8)1$	

10. A right circular cone of height  $h$  cm is inscribed in a sphere of radius 10 cm as shown in the figure below.



(a) Find an expression for the radius of the cone in terms of h.

(3mks)



Consider  $\triangle OBC$ ;

Let centre be O;

$$OB = OA$$

$$= 10 \text{ cm}$$

$$OC = (h - 10)$$

$$r^2 = (10)^2 - (h - 10)^2$$

$$r^2 = 100 - (h^2 - 20h + 100)$$

$$r^2 = 100 - h^2 + 20h - 100$$

$$r^2 = -h^2 + 20h$$

$$r = \pm \sqrt{20h - h^2}.$$

(b) Find the value of h for which the volume of the cone is a maximum.

(3mks)

$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \pi \times (\sqrt{20h - h^2})^2 \times h$$

$$= \frac{1}{3} \pi (20h^2 - h^3)$$

$$= \frac{20}{3} \pi h^2 - \frac{1}{3} \pi h^3$$

At  $V_{\max}$ ;

$$\frac{dV}{dh} = 0$$

$$\frac{40}{3} \pi h = \pi h^2 = 0$$

$$\pi h \left( \frac{40}{3} - h \right) = 0$$

$$\pi h = 0,$$

$$h = 0$$

But  $h \neq 0$

$$\therefore \frac{40}{3} - h = 0$$

$$h = \frac{40}{3}$$

$$= 10 \frac{1}{3} \text{ cm.}$$

- (c) Calculate in terms of  $\pi$ , the difference between the volume of the sphere and largest possible volume of the cone. (4mks)

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \pi \times (10)^3$$

$$= \frac{4000}{3} \pi \text{ cm}^3$$

$$V_{\text{max}} = \frac{1}{3} \pi r_{\text{max}} h_{\text{max}}$$

$$r = \pm \sqrt{20 \left(13 \frac{1}{3}\right) - \left(13 \frac{1}{3}\right)^2}$$

$$r^2 = \frac{800}{3} - \frac{1600}{9}$$

$$= \frac{800}{9}$$

$$V_{\text{max}} = \frac{1}{3} \pi \times \frac{800}{9} \times \frac{40}{3}$$

$$= \frac{32000}{81}$$

Difference in volume;

$$= \frac{4000}{3} - \frac{32000}{81}$$

$$= 938.2716 \text{ cm}^3.$$

# Kinematics.

1. A particle moves in a straight line such that the displacement  $S = 2t^3 - 5t^2 + 4t + 3$ , where  $t$  is time in seconds. Find;

- (a) The displacement of the particle at  $t = 4$  second. (2mks)

$$S = 2t^3 - 5t^2 + 4t + 3$$

$$\text{at } t = 4 \text{ seconds;}$$

$$S = 2(4)^3 - 5(4)^2 + 4(4) + 3$$

$$= 67 \text{ m.}$$

- (b) The velocity of the particle when  $t = 2$  seconds. (3mks)

$$v = \frac{dS}{dt}$$

$$v = 6t^2 - 10t + 4$$

$$\text{at } t = 2 \text{ seconds;}$$

$$v = 6(2)^2 - 10(2) + 4$$

$$= 8 \text{ m/s.}$$

- (c) The values of  $t$  when the particle is momentarily at rest. (3mks)

**Momentarily at rest;**

$$v = 0$$

$$6t^2 - 10t + 4 = 0$$

$$t = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(6)(4)}}{2(6)}$$

$$= \frac{10 \pm 2}{12}$$

$$t = 1 \text{ second}$$

or

$$t = \frac{2}{3} \text{ second.}$$

- (d) The acceleration of the particle when  $t = 1$  seconds. (2mks)

$$a = \frac{dv}{dt}$$

$$a = 12t - 10$$

$$\text{at } t = 1 \text{ second;}$$

$$a = 12(1) - 10$$

$$= 2 \text{ m/s}^2.$$

2. The displacement  $S$  metres of a moving particle after  $t$  seconds is given by  $S = 2t^3 - t^2 - 4t$ . Determine;

(a) The velocity of the particle when  $t = 3$  seconds. (3mks)

$$S = 2t^3 - t^2 - 4t$$

$$v = \frac{dS}{dt}$$

$$V = 6t^2 - 2t - 4$$

at  $t = 3$  seconds;

$$\begin{aligned} v &= 6(3)^2 - 2(3) - 4 \\ &= 44 \text{ m/s.} \end{aligned}$$

(b) The value of  $t$  when the particle is momentarily at rest. (3mks)

**Momentarily at rest;**

$$V = 0$$

$$6t^2 - 2t - 4 = 0$$

$$\begin{aligned} t &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(6)(-4)}}{2(6)} \\ &= \frac{2 \pm 10}{12} \end{aligned}$$

$t = 1$  second

or

$$t = -\frac{2}{3} \text{ second}$$

$$\text{But } t \neq -\frac{2}{3} \text{ second}$$

$$\therefore t = 1 \text{ second.}$$

(c) The displacement when the particle is momentarily at rest. (2mks)

**The particle is momentarily at rest;**

at  $t = 1$  second

$$S = 2(1)^3 - (1)^2 - 4(1)$$

$$= -3 \text{ m}$$

**Displacement is 3 m below starting point.**

(d) The acceleration of the particle when  $t = 2$  seconds. (2mks)

$$a = \frac{dV}{dt}$$

$$a = 12t - 2$$

at  $t = 2$  second;

$$a = 12(2) - 2$$

$$= 22 \text{ m/s}^2.$$

3. A particle moves in a straight line from a fixed point. Its velocity  $V$  m/s after  $t$  seconds is given by  $V = 4t^2 - 5t + 1$ . Find;

(a) The acceleration of the particle after 2 seconds. (2mks)

$$V = 4t^2 - 5t + 1$$

$$a = \frac{dV}{dt}$$

$$a = 8t - 5$$

at  $t = 2$  second;

$$a = 8(2) - 5$$

$$= 11 \text{ m/s}^2.$$

(b) The distance travelled during the third second. (3mks)

$$s = \int_2^3 (4t^2 - 5t + 1) dt$$

$$= \left[ \frac{4}{3}t^3 - \frac{5}{2}t^2 + t \right]_2^3$$

$$= \left[ \frac{4}{3}(3)^3 - \frac{5}{2}(3)^2 + (3) \right] - \left[ \frac{4}{3}(2)^3 - \frac{5}{2}(2)^2 + (2) \right]$$

$$= 16\frac{1}{2} - 2\frac{2}{3}$$

$$= 13\frac{5}{6} \text{ m.}$$

(c) The time when;

(i) The particle will be momentarily at rest. (3mks)

**Momentarily at rest;**

$$V = 0$$

$$4t^2 - 5t + 1 = 0$$

$$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(1)}}{2(4)}$$

$$= \frac{5 \pm 3}{8}$$

$$t = 1 \text{ second}$$

or

$$t = \frac{1}{4} \text{ second}$$

(ii) The particle will attain minimum velocity. (2mks)

**At minimum velocity;**

$$a = \frac{dV}{dt} = 0$$

$$8t - 5 = 0$$

$$t = \frac{5}{8} \text{ second.}$$

4. A particle moves along path OR such that  $t$  seconds after leaving point O, its velocity  $V$  m/s is given by  $V = 4t^2 - kt + 9$ , where  $k$  is a constant. The particle is momentarily at rest at R when the time,  $t = 1.5$  seconds. Determine;

(a) The value of  $k$ . (2mks)

$V = 4t^2 - kt + 9$ <p style="text-align: center;"><b>Momentarily at rest;</b></p> $V = 0$ <p style="text-align: center;"><b>at <math>t = 1.5</math> seconds;</b></p>		$4(1.5)^2 - k(1.5) + 9 = 0$ $18 - 1.5k = 0$ $1.5 = 18$ $k = 12.$
---	--	--

(b) The distance from O to R. (3mks)

$S = \int_0^{1.5} (4t^2 - 12t + 9) dt$ $= \left[ \frac{4}{3}t^3 - 6t^2 + 9t \right]_0^{1.5}$ $= \left[ \frac{4}{3}(1.5)^3 - 6(1.5)^2 + 9(1.5) \right] - \left[ \frac{4}{3}(0)^3 - 6(0)^2 + 9(0) \right]$		$= 4\frac{1}{2} - 0$ $= 4\frac{1}{2} \text{ m.}$
--	--	--

(c) Find the distance covered in the third second. (3mks)

<p><b>Distance in the 3<sup>rd</sup> second;</b></p> <p><b>Is the distance between;</b></p> <p style="text-align: center;"><b><math>t = 2</math> seconds</b></p> <p style="text-align: center;"><b>and <math>t = 3</math> seconds</b></p> $S = \int_2^3 (4t^2 - 12t + 9) dt$		$= \left[ \frac{4}{3}t^3 - 6t^2 + 9t \right]_2^3$ $= \left[ \frac{4}{3}(3)^3 - 6(3)^2 + 9(3) \right] - \left[ \frac{4}{3}(2)^3 - 6(2)^2 + 9(2) \right]$ $= 9 - 4\frac{2}{3}$ $= 4\frac{1}{3} \text{ m.}$
--	--	--

(d) The particle's acceleration when  $t = 1$  second.

(2mks)

$$a = \frac{dV}{dt}$$
$$a = 8t - 12.$$

**at  $t = 1$  second;**

$$a = 8(1) - 12$$
$$= -4 \text{ m/s}^2.$$

5. The acceleration of a body moving along a straight line is  $(4 - t) \text{ m/s}^2$  and its velocity is  $V \text{ m/s}$  after  $t$  seconds.

(a) If the initial velocity of the body is  $3 \text{ m/s}$ ;

(i) Express the velocity  $v$  in terms of  $t$ .

(3mks)

$$a = (4 - t) \text{ m/s}^2$$

$$V = \int (4 - t) dt$$

$$V = 4t - \frac{1}{2}t^2 + c$$

**At initial velocity;**

$$V = 3 \text{ m/s}, t = 0\text{s}$$

$$3 = 4(0) - \frac{1}{2}(0)^2 + c$$

$$c = 3$$

$$V = 4t - \frac{1}{2}t^2 + 3$$

(ii) Find the velocity of the body after 2 seconds.

(2mks)

**at  $t = 2 \text{ s}$ ;**

$$V = 4(2) - \frac{1}{2}(2)^2 + 3$$

$$= 9 \text{ m/s.}$$

(b) Calculate;

(i) The time taken to attain the maximum velocity.

(2mks)

**At maximum velocity;**

$$a = \frac{dV}{dt} = 0$$

$$4 - t = 0$$

$$t = 4 \text{ seconds.}$$

(ii) The distance covered by the body to attain the maximum velocity. (3mks)

**Distance covered to attain;**

**Maximum Velocity is |b|;**

**t = 0 seconds**

**and t = 4 seconds**

$$S = \int_0^4 \left( 4t - \frac{1}{2}t^2 + 3 \right) dt$$

$$= \left[ 2t^2 - \frac{1}{6}t^3 + 3t \right]_0^4$$

$$= \left[ 2(4)^2 - \frac{1}{6}(4)^3 + 3(4) \right] - \left[ 2(0)^2 - \frac{1}{6}(0)^3 + 3(0) \right]$$

$$= 33\frac{1}{3} - 0$$

$$= 33\frac{1}{3} \text{ m.}$$

6. A particle moves along a straight line such that its displacement, S(m) from a given point is  $S = t^3 - 3t^2 + 5$ , where t is time in seconds. Find;

(a) The displacement of the particle during the 4<sup>th</sup> second. (2mks)

**Distance during 4<sup>th</sup> second;**

**Is the distance between;**

**t = 3 seconds**

**and t = 4 seconds**

$$= [t^3 - 3t^2 + 5]_3^4$$

$$= [(4)^3 - 3(4)^2 + 5] - [(3)^3 - 3(3)^2 + 5]$$

$$= 21 - 5$$

$$= 16 \text{ m.}$$

(b) The velocity of the particle when t = 5 seconds. (3mks)

$$v = \frac{ds}{dt}$$

$$= 3t^2 - 6t$$

**at t = 5 seconds;**

$$v = 3(5)^2 - 6(5)$$

$$= 45 \text{ m/s.}$$

(c) The values of t when the particle is momentarily at rest. (3mks)

**Momentarily at rest;**

$$\frac{ds}{dt} = v = 0$$

$$3t^2 - 6t = 0$$

$$3t(t - 2) = 0$$

$$3t = 0$$

**t = 0 seconds**

**or**

**t = 2 seconds.**

(d) The acceleration of the particle when  $t = 2$  seconds.

(2mks)

$$a = \frac{dV}{dt}$$
$$a = 6t - 6.$$

at  $t = 2$  seconds;

$$a = 6(2) - 6$$
$$= 6 \text{ m/s}^2.$$

7. A particle moving in a straight line is such that its displacement  $S$  metres from a point is  $S = t^3 - 6t^2 + 2t + 3$  where  $t$  is time in seconds. Find;

(a) The displacement of the particle at  $t = 3$  seconds.

(2mks)

$$S = t^3 - 6t^2 + 2t + 3$$

at  $t = 3$  seconds;

$$S = (3)^3 - 6(3)^2 + 2(3) + 3$$
$$= -18 \text{ m}$$

Distance is 18 m below starting point.

(b) The velocity of the particle at  $t = 4$  seconds.

(2mks)

$$v = \frac{dS}{dt}$$
$$= 3t^2 - 12t + 2$$

at  $t = 4$  seconds;

$$v = 3(4)^2 - 12(4) + 2$$
$$= 2 \text{ m/s.}$$

(c) The value of  $t$  when the particle is momentarily at rest.

(3mks)

Momentarily at rest;

$$v = 0$$

$$3t^2 - 12t + 2 = 0$$

$$t = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{12 \pm \sqrt{120}}{6}$$

$$t = 3.826 \text{ seconds}$$

or

$$t = 0.1743 \text{ seconds}$$

(d) The minimum velocity attained by the particle.

(3mks)

**At minimum velocity;**

$$a = \frac{dV}{dt} = 0$$

$$6t - 12 = 0$$

**t = 2 seconds**

**Particle has minimum velocity at t = 2 s**

$$V = 3(2)^2 - 12(2) + 2$$

$$= -10 \text{ m/s.}$$

8. The velocity  $V$  m/s of a particle projected into space is given by the formula  $V = 4t^2 - 2t + 9$ , where  $t$  is time in seconds. Determine;

(a) The acceleration of the particle when  $t = 2$  seconds.

(3mks)

$$V = 4t^2 - 2t + 9$$

$$a = \frac{dV}{dt}$$

$$a = 8t - 2$$

**at t = 2 seconds;**

$$a = 8(2) - 2$$

$$= 14 \text{ m/s}^2.$$

(b) The value of  $t$  when the velocity is minimum.

(2mks)

**At minimum velocity;**

$$a = \frac{dV}{dt} = 0$$

$$8t - 2 = 0$$

$$t = \frac{1}{4} \text{ seconds.}$$

(c) The minimum velocity attained by the particle.

(2mks)

**$V_{\min}$  is at  $t = \frac{1}{4}$  seconds;**

$$V_{\min} = 4\left(\frac{1}{4}\right)^2 - 2\left(\frac{1}{4}\right) + 9$$

$$V_{\min} = 8\frac{3}{4} \text{ m/s.}$$

(d) The distance covered during the 2<sup>nd</sup> second.

(3mks)

**Distance during 2<sup>nd</sup> second;**

**Is the distance between;**

**t = 1 seconds**

**and t = 2 seconds**

$$= \int_1^2 (4t^2 - 2t + 9) dt.$$

$$= \left[ \frac{4}{3} t^3 - t^2 + 9t \right]_1^2$$

$$= \left[ \frac{4}{3} (2)^3 - (2)^2 + 9(2) \right] - \left[ \frac{4}{3} (1)^3 - (1)^2 + 9(1) \right]$$

$$= 24 \frac{2}{3} - 9 \frac{1}{3}$$

$$= 15 \frac{1}{3} \text{ m.}$$

9. The acceleration of a particle t seconds after passing a fixed point P is given by  $a = 3t - 3$ . Given that the velocity of the particle when  $t = 2$  seconds is 5 m/s. Find;

(a) Its velocity;

(i) In terms of t.

(3mks)

$$a = 3t - 3$$

$$V = \int (3t - 3) dt$$

$$V = \frac{3}{2} t^2 - 3t + c$$

**when t = 2 seconds, V = 5 m/s**

$$\frac{3}{2} (2)^2 - 3(2) + c = 5$$

$$c = 5$$

$$V = \frac{3}{2} t^2 - 3t + 5$$

(ii) When t = 4 seconds.

(2mks)

$$V = \frac{3}{2} t^2 - 3t + 5$$

**at t = 4 seconds;**

$$V = \frac{3}{2} (4)^2 - 3(4) + 5$$

$$= 17 \text{ m/s.}$$

(b) The maximum velocity attained by the particle.

(2mks)

**At maximum velocity;**

$$a = \frac{dV}{dt} = 0$$

$$3t - 3 = 0$$

**t = 1 second**

$$V_{\max} = \frac{3}{2}(1)^2 - 3(1) + 5$$

$$V_{\max} = 3\frac{1}{2} \text{ m/s.}$$

(c) Its displacement during the third second.

(3mks)

**Distance during 3<sup>rd</sup> second;**

**Is the distance between;**

**t = 2 seconds**

**and t = 3 seconds**

$$= \int_2^3 \left( \frac{3}{2}t^2 - 3t + 5 \right) dt.$$

$$= \left[ \frac{1}{2}t^3 - \frac{3}{2}t^2 + 5t \right]_2^3$$

$$= \left[ \frac{1}{2}(3)^3 - \frac{3}{2}(3)^2 + 5(3) \right] - \left[ \frac{1}{2}(2)^3 - \frac{3}{2}(2)^2 + 5(2) \right]$$

$$= 15 - 8$$

$$= 7 \text{ m.}$$

10. A particle moves in a straight line so that t seconds after passing through a fixed point O, its velocity V m/s is given by the equation  $V = 20t - 2t^2$ . Find;

(a) Velocity at t = 6 seconds.

(2mks)

$$V = 20t - 2t^2$$

**at t = 6 seconds;**

$$V = 20(6) - 2(6)^2$$

$$= 48 \text{ m/s.}$$

(b) The initial acceleration of the particle.

(2mks)

**At initial acceleration;**

**t = 0**

$$a = \frac{dV}{dt}$$

$$a = 20 - 4t$$

**at t = 0**

$$a = 20 - 4(0)$$

$$= 20 \text{ m/s}^2.$$

(c) The maximum velocity attained by the particle.

(3mks)

**At maximum velocity;**

$$a = \frac{dv}{dt} = 0$$

$$20 - 4t = 0$$

**t = 5 seconds**

$$V_{\max} = 20(5) - 2(5)^2$$

$$V_{\max} = 50 \text{ m/s.}$$

(d) The displacement of the particle during the 5<sup>th</sup> second.

(3mks)

**Distance during 5<sup>th</sup> second;**

**Is the distance between;**

**t = 4 seconds**

**and t = 5 seconds**

$$= \int_4^5 (20t - 2t^2) dt.$$

$$= \left[ 10t^2 - \frac{2}{3}t^3 \right]_4^5$$

$$= \left[ 10(5)^2 - \frac{2}{3}(5)^3 \right] - \left[ 10(4)^2 - \frac{2}{3}(4)^3 \right]$$

$$= 166\frac{2}{3} - 117\frac{1}{3}$$

$$= 49\frac{1}{3} \text{ m.}$$